Understanding the Nature of CER and Cost Driver Uncertainty

OS 4012

Nature of CER and Cost Driver Uncertainty

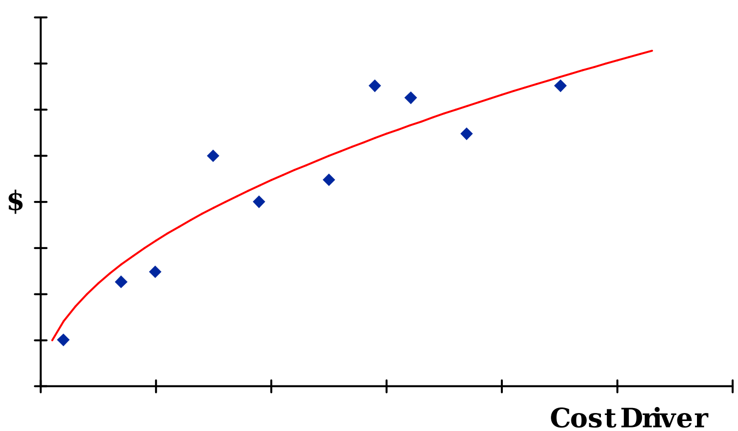
- At the end of this presentation we will have covered:
  - Sources of CER uncertainty
  - Sources of cost driver uncertainty
  - Other types of uncertainty
  - Prediction error vs. standard error
  - Subjective probability assessment

Introduction

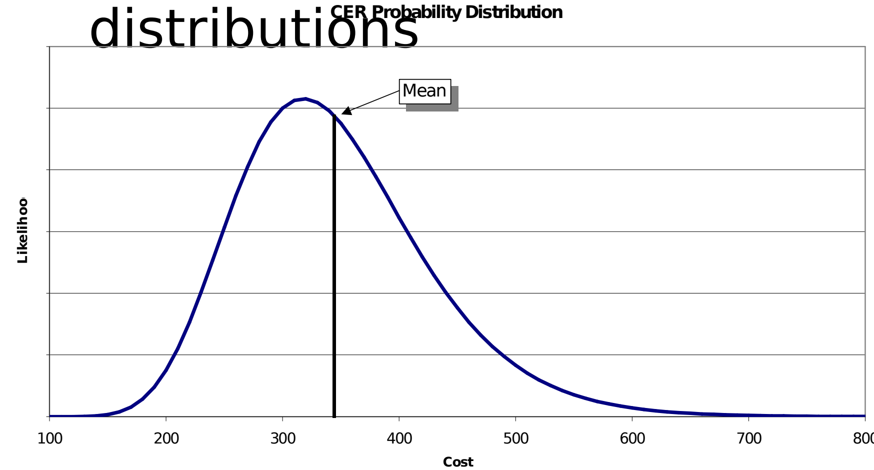
- Major sources of uncertainty in a cost estimate include:
  - Cost model uncertainty
  - Cost driver uncertainty
  - Other areas of uncertainty
    - Programmatic uncertainty
    - Technical uncertainty
    - Requirements uncertainty
- Ideally, we want to include all types of uncertainty in our cost estimates

Cost Model Uncertainty

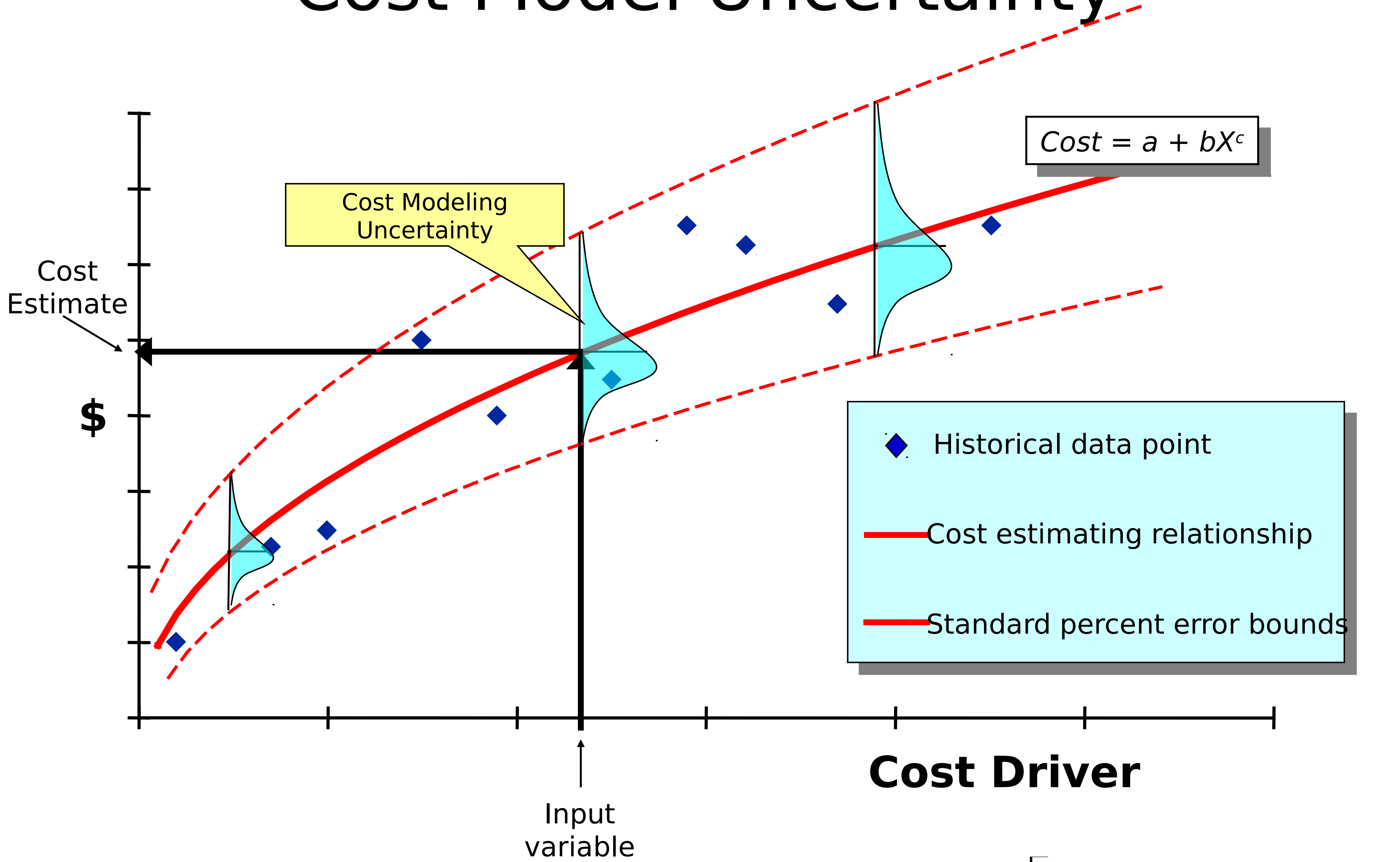
- Cost estimating relationships (CERs) are uncertain
- They have to be, since they are derived using regression!
- Typical CERs are based on historical cost data



- So, typical cost estimates have probability distributions



Cost Model Uncertainty



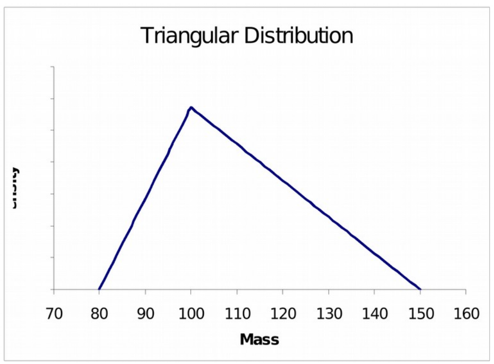
Cost Driver Uncertainty

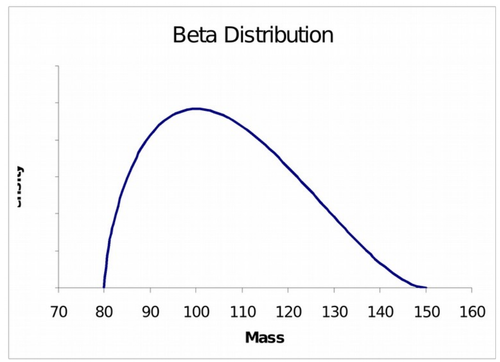
- Cost driver uncertainty is the uncertainty associated with cost modeling inputs
- Example:
  - Cost = $54+3(\text{mass})^{0.863}$ with some random error
  - If mass is 100 kg, then Cost = \$213.6 with some random error
  - But, what if the mass has uncertainty also?
    - 80 kg? 100 kg? 150 kg?
- The uncertainty of the cost modeling input plays a key role in the overall cost uncertainty

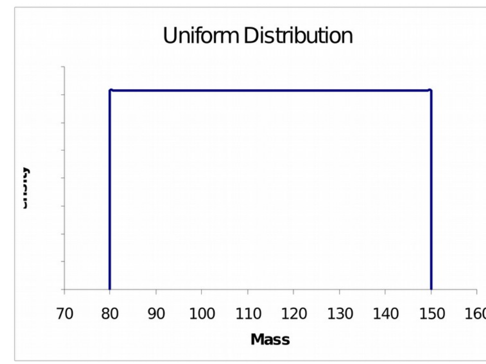
How Do We Portray Cost Driver Uncertainty?

- It is usually helpful to think of the cost driver value as a random variable itself

Triangular? **Beta?** **Uniform?**

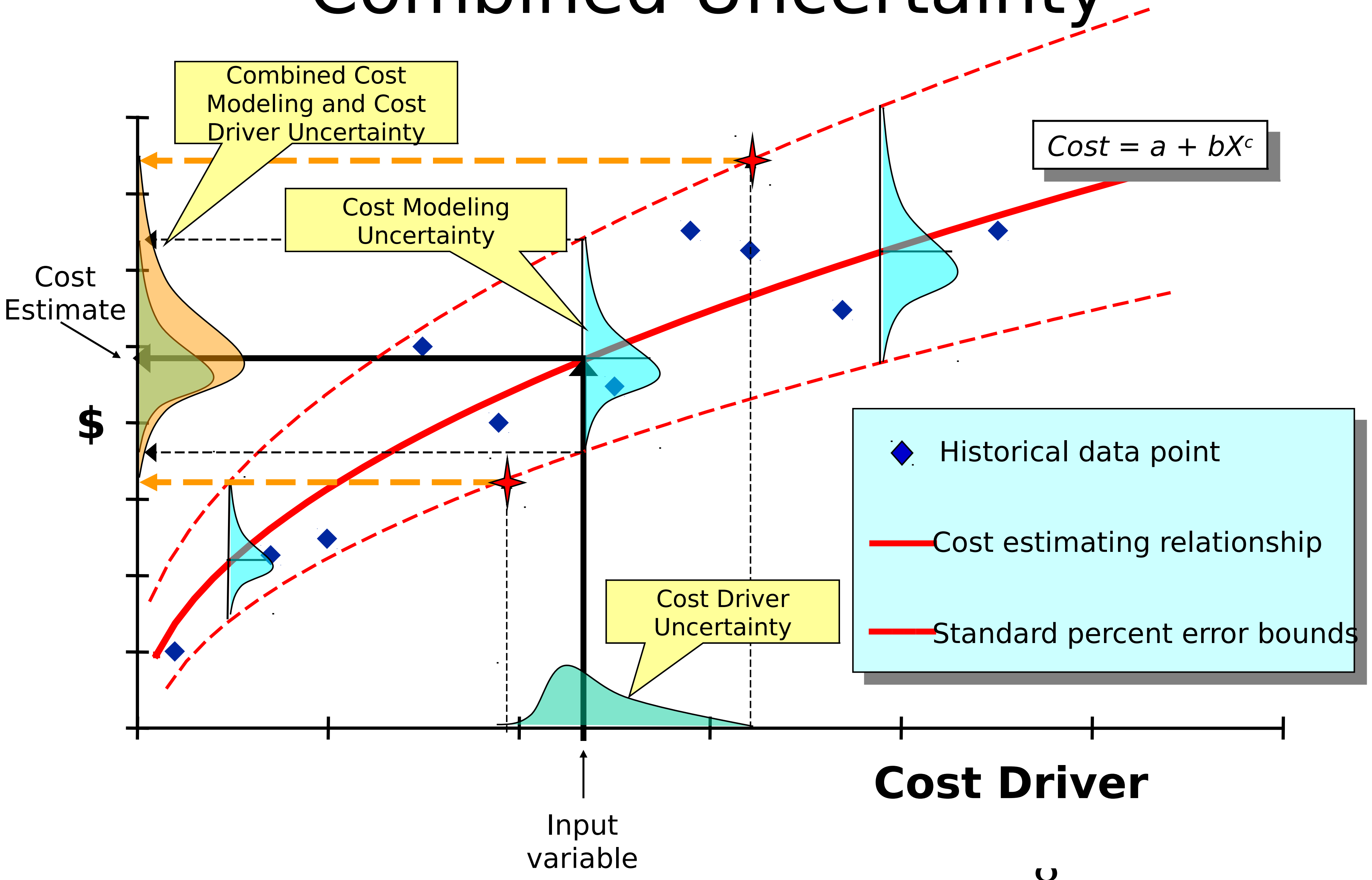






- The choice of the shape depends on the nature of the uncertainty
- Then, we can model the cost driver uncertainty in our Monte Carlo simulation in addition to the CER uncertainty

Combined Uncertainty



Programmatic Uncertainty

- Programmatic uncertainty is usually a result of schedule and/or funding uncertainty
- Program offices have master schedules for their programs
  - If the schedule slips, extra cost is usually incurred because a “standing army” of contractors will have to paid in order to keep them current with the contract
  - If the schedule is compressed, extra cost is usually incurred because contractors have to work overtime, or important events, such as testing, have to be reduced, leading to potential problems downstream
- Funding perturbations also leads to problems
  - If funding is decreased, important events have to be curtailed
  - If extra funding is forced on the contractors, they will find ways to spend it – and not necessarily on what is really needed

Technical Uncertainty

- With high technology acquisitions, there is often uncertainty that technology can be matured quickly enough to go operational when it is needed
  - E.g., a new system needs a “flux capacitor”
  - But “flux capacitors” may not have been invented yet
  - Perhaps a prototype exists somewhere, but it needs to be “matured”
- If the technology cannot be matured on time, this may lead to
  - Redesign
  - Delays
  - Lack of performance

Requirements Uncertainty

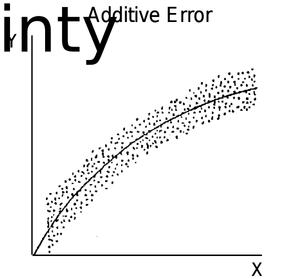
- Few DoD programs make it from Authority to Proceed (ATP) to Delivery with their initial set of requirements intact
  - Requirements changes occur frequently over the acquisition lifecycle
    - Sometimes for good reasons!
    - But not always
- When requirements change, the program office and contractor have to go “back to the drawing board” in order to make the system changes necessary to meet the new requirement
  - This can be expensive…involving additional systems engineering, leading to schedule slips and funding perturbations!

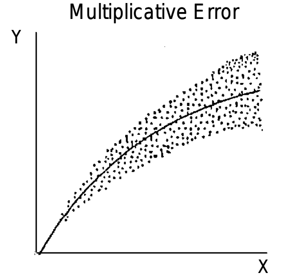
Modeling the Other Areas of Uncertainty

- It is difficult to model these other areas of uncertainty using Monte Carlo simulation
  - Programmatic
  - Technical
  - Requirements
- It’s not impossible, but it requires one to know ahead of time which events will occur!
  - And no program office wants to hear, “Your cost estimate is $X, and you will experience an additional $Y in programmatic, technical and requirements uncertainty”
- Research is currently underway to derive planning factors for these types of events though

CER Uncertainty

- Since CERs are usually the result of regression analysis, they also carry uncertainty
  - $Y = aX + \varepsilon$ $Y = aX \bullet \varepsilon$
  - $Y = a + bX + \varepsilon$ $Y = (a + bX) \bullet \varepsilon$
  - $Y = aX^{b} + \varepsilon$ $Y = aX^{b} \bullet \varepsilon$
  - $Y = a + bX^{c} + \varepsilon$ $Y = (a + bX^{c}) \bullet \varepsilon$

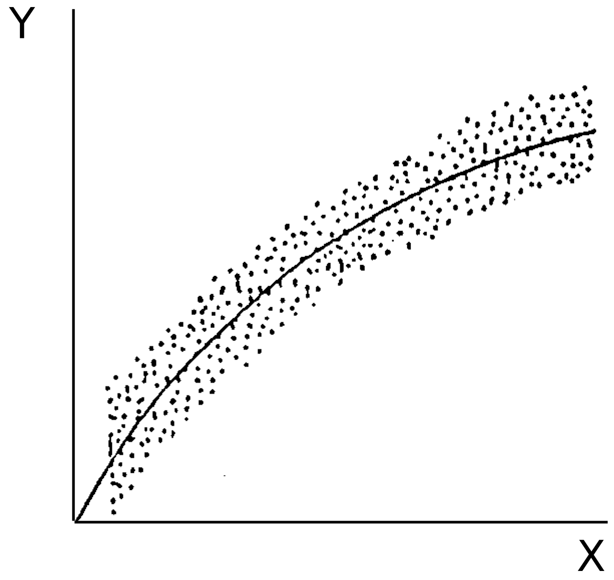




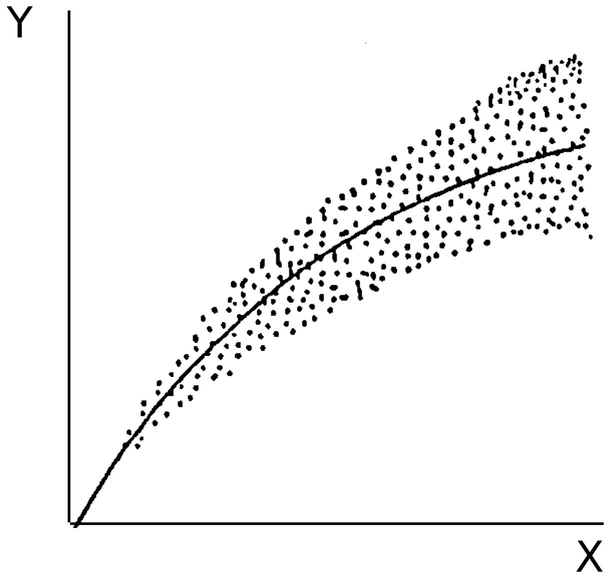
- where ε has some probability distribution with mean zero (in the additive error case) or 1.0 (in the multiplicative case), and some constant variance
- Regression analysis techniques are used to determine the characteristics of the curve, and also the variance

Error of Estimation

Additive Error



Multiplicative Error



Reference: H.L. Eskew and K.S. Lawler, "Correct and Incorrect Error Specifications in Statistical Cost Models," Journal of Cost Analysis, Spring 1994, page 107.

Standard Error of the Estimate

- The standard error of the estimate *(SE or SPE)* is the root mean square of all errors or percentage errors made in estimating points of the data base.
  - A "one-sigma" number that can be used to bound actual cost within an interval surrounding the estimate.

$$SPE = 100\% \times \sqrt{\frac{1}{n-m}\sum_{i=1}^{n}\left[\frac{y_i - f(x_i)}{f(x_i)}\right]^2}$$

(multiplicative-error model)

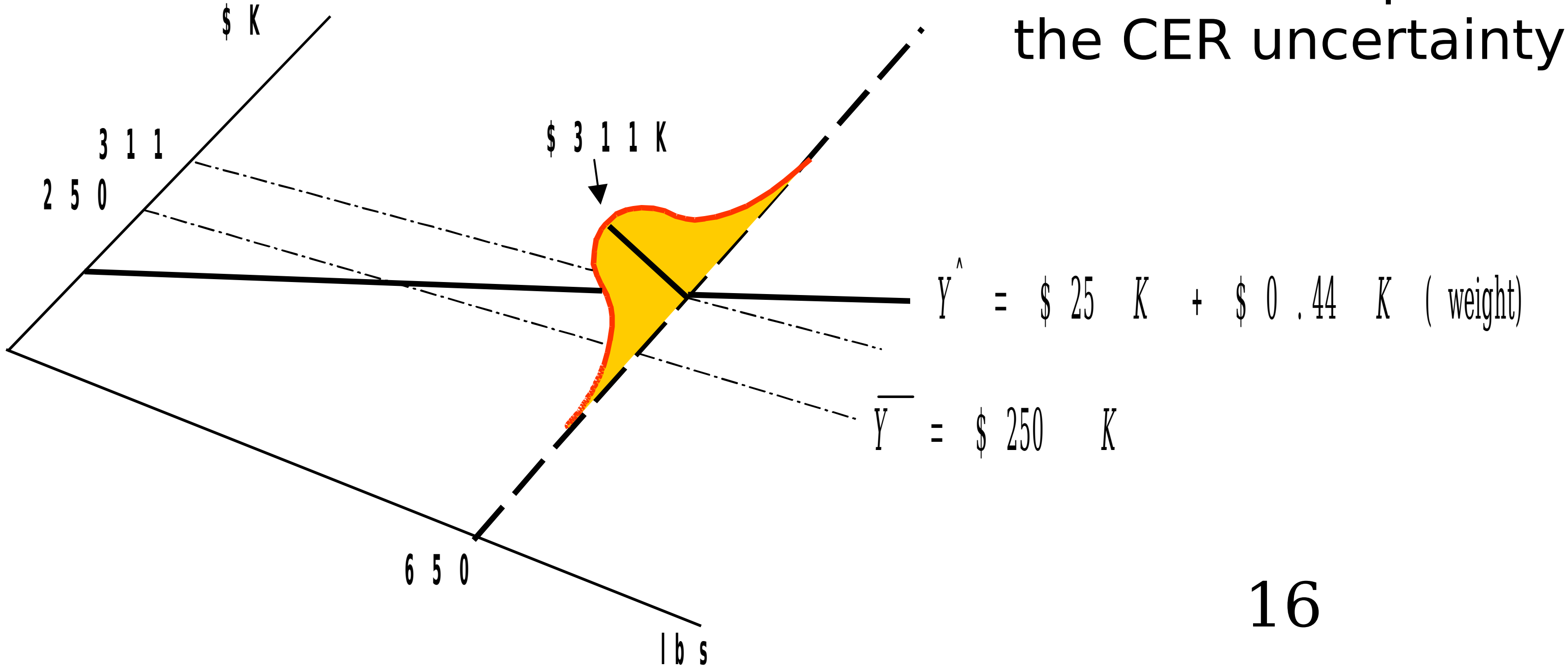
$$SE = \sqrt{\frac{1}{n-m}\sum_{i=1}^{n}\left[y_i - f(x_i)\right]^2}$$

(additive-error model)

  - Where *m* = number of fit parameters estimated in the model.

The Uncertainty of the CER

- The uncertainty of the CER is measured by the standard error (*SE*) or standard percent error (*SPE*)
- When evaluating a CER, we are actually "slicing through" a "probability-shaped tunnel" floating over the data
- The "width" of the "slice" corresponds to the CER uncertainty



Standard Error vs. Prediction Error

- The **standard error** (or **standard percent error**) is related to the uncertainty associated with the CER's ability to estimate its own data points
- The **prediction error**, on the other hand, is related to the CER's ability to predict a *new* data point
  - e.g., the one we are trying to estimate!
- Prediction errors are more uncertain than standard errors!
  - Because we have chosen fit parameters that make the standard error as small as possible
  - But, when we turn around and USE the CER to estimate the cost of something that is not in the CER database, more uncertainty exists!
- Issue: Prediction errors have been defined only for OLS regression fits!
  - Research is ongoing to define prediction errors for non-OLS regression equations

Confidence Intervals for an OLS CER

- For a typical single-variable OLS-derived linear CER, the (1-α) **confidence interval** for $\beta_0 + \beta_1 x^*$, the expected value of the cost estimate for a particular value x^* of the cost driver, is

$$\hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\alpha/2,\,n-2} \times s.e.\left(\hat{\beta}_0 + \hat{\beta}_1 x^*\right)$$

where

$$s.e.\left(\hat{\beta}_0 + \hat{\beta}_1 x^*\right) = \hat{\sigma}\sqrt{\frac{1}{n} + \frac{\left(x^* - \bar{x}\right)^2}{S_{XX}}} = SE\sqrt{\frac{1}{n} + \frac{\left(x^* - \bar{x}\right)^2}{S_{XX}}}$$

andwhere

$$S_{XX} = \sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \quad \text{and } SE = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i\right)\right)^2}{n-2}}$$

Additional Uncertainty When Predicting

- When one makes inferences about a future response at x^*, there are two sources of uncertainty
  - The first is the uncertainty in the value of the regression line at x^*
  - The second is the variability in the error term
- Thus, while the value on the fitted regression line $\beta_0 + \beta_1 x^*$ serves as a point estimate of the expected response at x^*, the total variability is

$$Var\left(\hat{\beta}_0 + \hat{\beta}_1 x^*\right) + \sigma^2$$

$$= \sigma^2 \left[\frac{1}{n} + \frac{\left(x^* - \bar{x}\right)^2}{S_{XX}} \right] + \sigma^2 = \sigma^2 \left[\frac{n+1}{n} + \frac{\left(x^* - \bar{x}\right)^2}{S_{XX}} \right]$$

which is the variance of the fitted regression line together with the variance of the error term

Prediction Intervals for an OLS CER

- So, for our single-variable OLS-derived linear CER, the (1-α) **prediction interval** for $\beta_0 + \beta_1 x^*$, the expected **future** value of the cost estimate for a particular value x^* of the cost driver, is

$$\hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\alpha/2, n-2} \times \hat{\sigma} \sqrt{\frac{n+1}{n} + \frac{\left(x^* - \bar{x}\right)^2}{S_{XX}}}$$

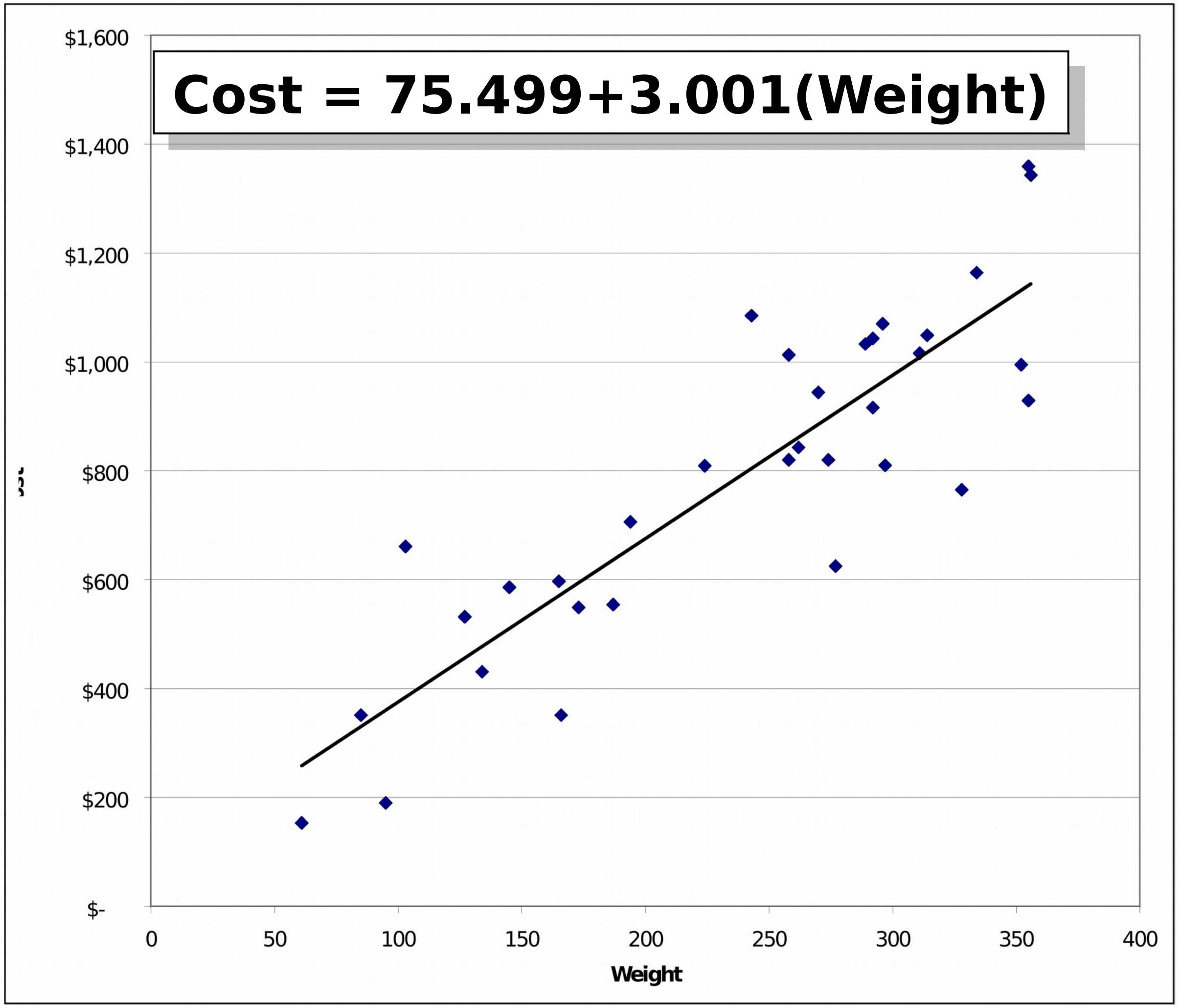
$$= \hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\alpha/2, n-2} \times SE \sqrt{\frac{n+1}{n} + \frac{\left(x^* - \bar{x}\right)^2}{S_{XX}}}$$

where

$$S_{XX} = \sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2 \quad \text{and } SE = \sqrt{\frac{\sum_{i=1}^{n} \left(y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i\right)\right)^2}{n-2}}$$

Example

Cost (Y)	Weight (X)
$ 153	61
$ 351	85
$ 190	95
$ 661	103
$ 532	127
$ 431	134
$ 586	145
$ 597	165
$ 351	166
$ 549	173
$ 554	187
$ 706	194
$ 809	224
$ 1,085	243
$ 1,013	258
$ 820	258
$ 843	262
$ 944	270
$ 820	274
$ 625	277
$ 1,033	289
$ 1,043	292
$ 916	292
$ 1,070	296
$ 810	297
$ 1,016	311
$ 1,049	314
$ 765	328
$ 1,164	334
$ 995	352
$ 929	355
$ 1,359	355
$ 1,343	356



Regression Results

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.8738
R Square	0.7636
Adjusted R Squar	0.7560
Standard Error	149.30
Observations	33

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2231939.0	2231939.0	100.1	3.15E-11
Residual	31	690975.6	22289.5		
Total	32	2922914.5			

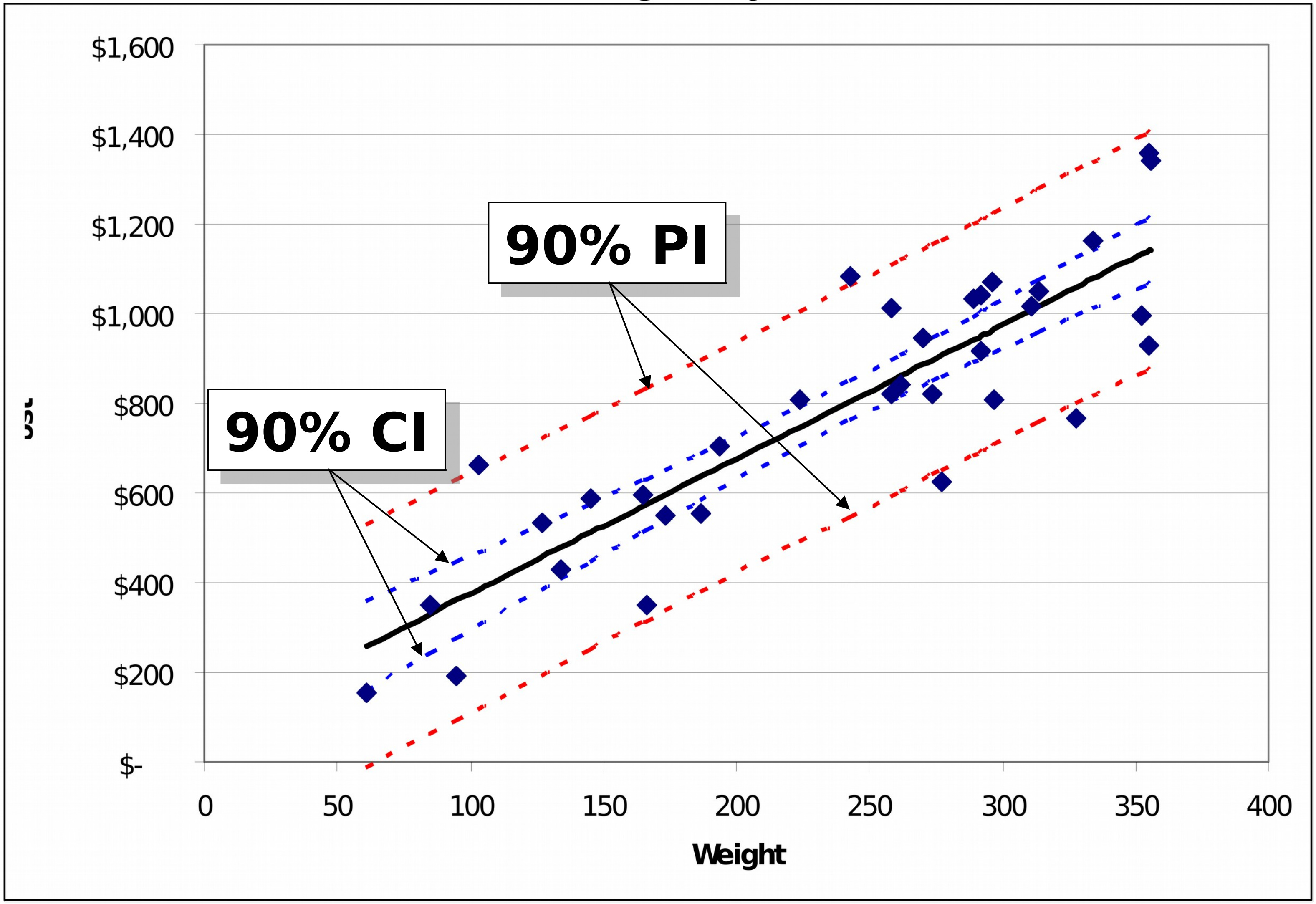
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	75.499	76.105	0.992	0.329
Weight (X)	3.001	0.300	10.007	0.000

Calculating CIs and PIs

Cost (Y)	Weight (X)	CER-CI	CER	CER+CI	CER-PI	CER	CER+PI
$ 153	61	$ 158.1	$ 258.5	$ 359.0	$ (13.8)	$ 258.5	$ 530.9
$ 351	85	$ 240.9	$ 330.5	$ 420.2	$ 62.0	$ 330.5	$ 599.1
$ 190	95	$ 275.3	$ 360.6	$ 445.8	$ 93.4	$ 360.6	$ 627.7
$ 661	103	$ 302.8	$ 384.6	$ 466.4	$ 118.5	$ 384.6	$ 650.6
$ 532	127	$ 384.8	$ 456.6	$ 528.4	$ 193.4	$ 456.6	$ 719.7
$ 431	134	$ 408.5	$ 477.6	$ 546.6	$ 215.2	$ 477.6	$ 740.0
$ 586	145	$ 445.7	$ 510.6	$ 575.4	$ 249.3	$ 510.6	$ 771.9
$ 597	165	$ 512.8	$ 570.6	$ 628.4	$ 310.9	$ 570.6	$ 830.2
$ 351	166	$ 516.1	$ 573.6	$ 631.1	$ 314.0	$ 573.6	$ 833.2
$ 549	173	$ 539.4	$ 594.6	$ 649.8	$ 335.5	$ 594.6	$ 853.7
$ 554	187	$ 585.3	$ 636.6	$ 687.9	$ 378.3	$ 636.6	$ 894.9
$ 706	194	$ 608.1	$ 657.6	$ 707.2	$ 399.7	$ 657.6	$ 915.5
$ 809	224	$ 702.9	$ 747.6	$ 792.3	$ 490.6	$ 747.6	$ 1,004.7
$ 1,085	243	$ 760.5	$ 804.6	$ 848.8	$ 547.7	$ 804.6	$ 1,061.6
$ 1,013	258	$ 804.5	$ 849.6	$ 894.8	$ 592.5	$ 849.6	$ 1,106.8
$ 820	258	$ 804.5	$ 849.6	$ 894.8	$ 592.5	$ 849.6	$ 1,106.8
$ 843	262	$ 816.0	$ 861.6	$ 907.3	$ 604.4	$ 861.6	$ 1,118.9
$ 944	270	$ 838.8	$ 885.7	$ 932.5	$ 628.2	$ 885.7	$ 1,143.1
$ 820	274	$ 850.0	$ 897.7	$ 945.3	$ 640.1	$ 897.7	$ 1,155.2
$ 625	277	$ 858.5	$ 906.7	$ 954.9	$ 649.0	$ 906.7	$ 1,164.3
$ 1,033	289	$ 891.7	$ 942.7	$ 993.7	$ 684.4	$ 942.7	$ 1,200.9
$ 1,043	292	$ 899.9	$ 951.7	$ 1,003.4	$ 693.3	$ 951.7	$ 1,210.0
$ 916	292	$ 899.9	$ 951.7	$ 1,003.4	$ 693.3	$ 951.7	$ 1,210.0
$ 1,070	296	$ 910.8	$ 963.7	$ 1,016.5	$ 705.1	$ 963.7	$ 1,222.3
$ 810	297	$ 913.5	$ 966.7	$ 1,019.8	$ 708.0	$ 966.7	$ 1,225.3
$ 1,016	311	$ 951.2	$ 1,008.7	$ 1,066.1	$ 749.1	$ 1,008.7	$ 1,268.2
$ 1,049	314	$ 959.3	$ 1,017.7	$ 1,076.1	$ 757.9	$ 1,017.7	$ 1,277.5
$ 765	328	$ 996.4	$ 1,059.7	$ 1,123.0	$ 798.8	$ 1,059.7	$ 1,320.6
$ 1,164	334	$ 1,012.1	$ 1,077.7	$ 1,143.2	$ 816.2	$ 1,077.7	$ 1,339.2
$ 995	352	$ 1,059.1	$ 1,131.7	$ 1,204.3	$ 868.4	$ 1,131.7	$ 1,395.0
$ 929	355	$ 1,066.9	$ 1,140.7	$ 1,214.5	$ 877.0	$ 1,140.7	$ 1,404.4
$ 1,359	355	$ 1,066.9	$ 1,140.7	$ 1,214.5	$ 877.0	$ 1,140.7	$ 1,404.4
$ 1,343	356	$ 1,069.5	$ 1,143.7	$ 1,217.9	$ 879.9	$ 1,143.7	$ 1,407.5

X-bar	238.5
S_{XX}	247898.2
SE	$ 149.3
α	10%
$1-\alpha$	90%
n	33
$t_{a/2,n-2}$	1.696

Confidence Interval vs. Prediction Interval



But What If We Are Not Using OLS-Derived CERs?

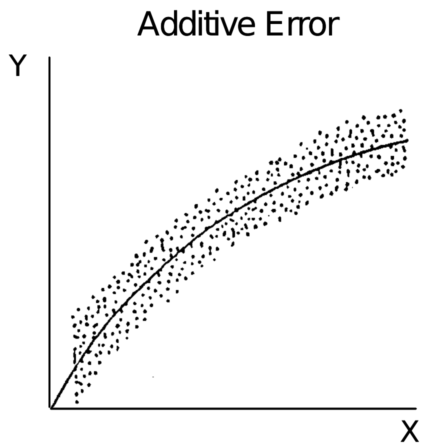
- If we are NOT using OLS-derived CERs
  - E.g… $Y=aX^b$
    $Y=a+bX^c$
- Then, we need to be more creative!
  - Not only do we NOT have the backing of OLS assumptions
  - But we also may be dealing with either ADDITIVE or MULTIPLICATIVE errors!
  - And, we usually assume the error specification is LOGNORMAL!
- So, when using non-OLS CERs, the usual regression assumptions do not apply
  - And, CI's and PI's have not been defined for non-OLS CERs
  - However, research is underway to develop these
- In the meantime, we can approximate CIs and PIs by applying the same ideas anyway (while awaiting someone's PhD thesis on this topic!)

Non-OLS CER Confidence Interval Approximation – ADDITIVE Errors

- Suppose we start with a CER of the form $Y=a+bX^c$, with ADDITIVE errors and a lognormal error specification
- Then we can assume that the same rules apply and run with it
  - What have we got to lose?
- E.g.,

$$Cost = -15.69 + 9.44\left(Weight\right)^{0.51}$$

$$SE = \$10.5$$



$$(1-\alpha)CI \approx a + b\left(x^*\right)^c \pm \left(\alpha/2 \text{ percentile}\right) \times s.e.\left(a + b\left(x^*\right)^c\right)$$

where

$$s.e.\left(a + b\left(x^*\right)^c\right) = \hat{\sigma}\sqrt{\frac{1}{n} + \frac{\left(x^* - \bar{x}\right)^2}{S_{XX}}} = SE\sqrt{\frac{1}{n} + \frac{\left(x^* - \bar{x}\right)^2}{S_{XX}}}$$

and where

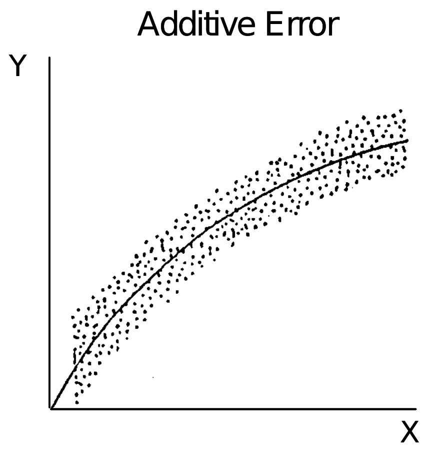
$$\left(\alpha/2 \text{ percentile}\right) \text{ is the } \alpha/2^{th} \text{ percentile of the lognormal error distribution}$$

Non-OLS CER Prediction Interval Approximation

- Then it is a simple extension to arrive at the Prediction Interval
- E.g.,

$$Cost = -15.60 + 9.44(Weight)^{0.51}$$

$$SE = \$10.5$$



$$(1-\alpha)PI \approx a + b(x^*)^c \pm \left(\tfrac{\alpha}{2}\text{ percentile}\right) \times s.e.\left(a + b(x^*)^c\right)$$

where

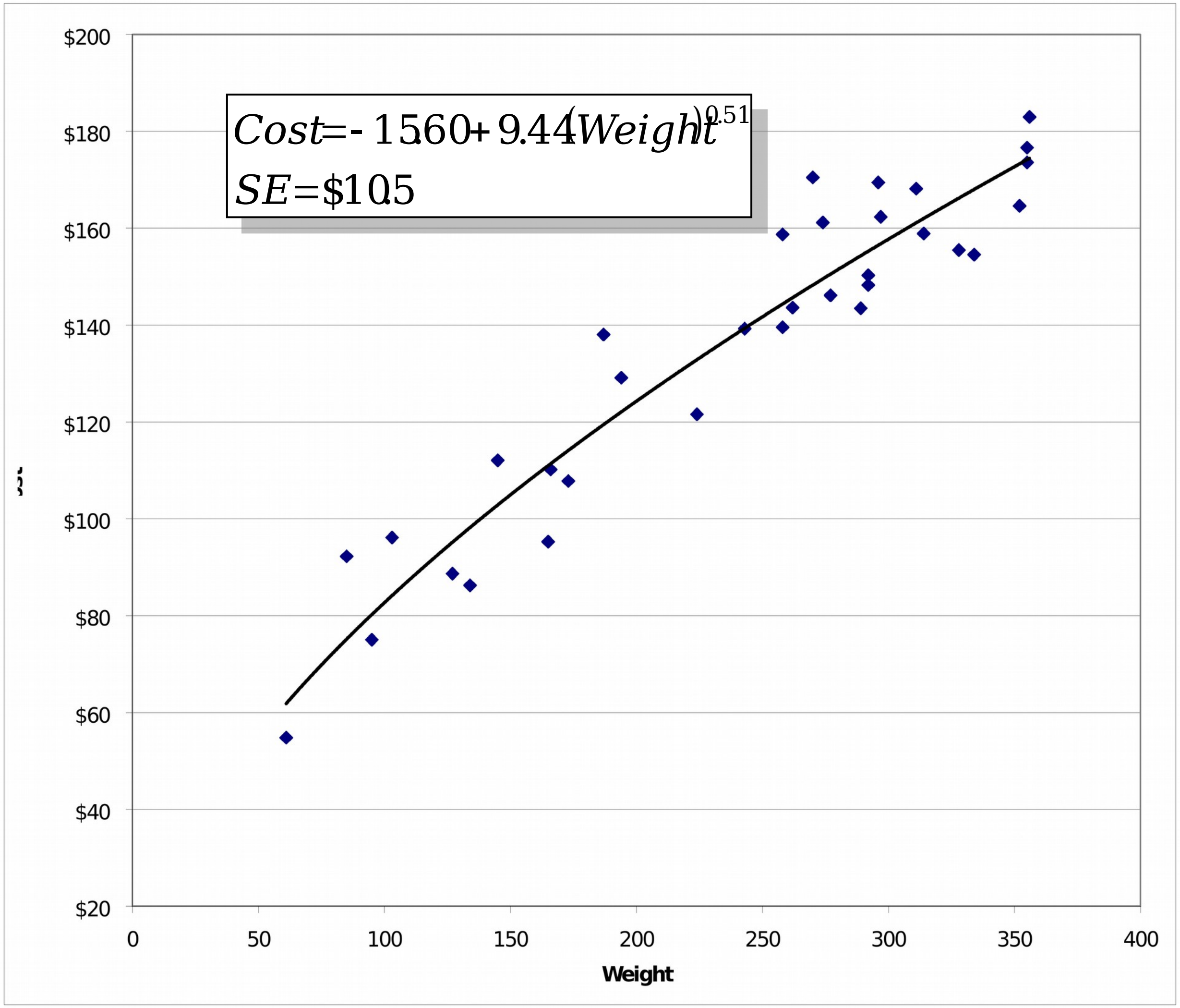
$$s.e.\left(a + b(x^*)^c\right) = \hat{\sigma}\sqrt{\frac{n+1}{n} + \frac{(x^* - \bar{x})^2}{S_{XX}}} = SE\sqrt{\frac{n+1}{n} + \frac{(x^* - \bar{x})^2}{S_{XX}}}$$

and where

$\left(\tfrac{\alpha}{2}\text{ percentile}\right)$ is the $\tfrac{\alpha}{2}^{th}$ percentile of the lognormal error distribution

Example (ADDITIVE Errors)

Cost (Y)	Weight (X)
$ 55	61
$ 92	85
$ 75	95
$ 96	103
$ 89	127
$ 86	134
$ 112	145
$ 95	165
$ 110	166
$ 108	173
$ 138	187
$ 129	194
$ 122	224
$ 139	243
$ 140	258
$ 159	258
$ 144	262
$ 170	270
$ 161	274
$ 146	277
$ 143	289
$ 148	292
$ 150	292
$ 169	296
$ 162	297
$ 168	311
$ 159	314
$ 156	328
$ 155	334
$ 165	352
$ 174	355
$ 177	355
$ 183	356



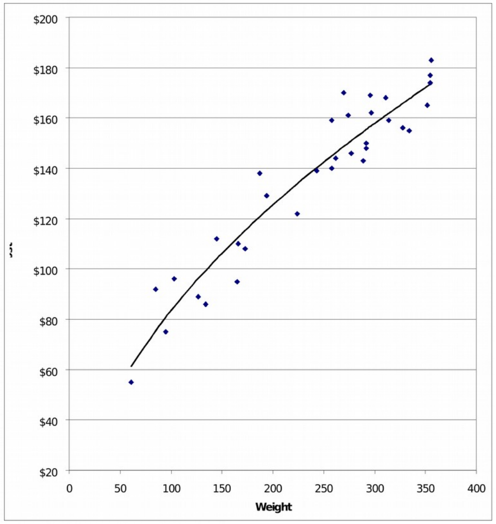
ADDITIVE GERM Regression Results

Cost (Y)	Weight (X)	CER	Y-CER	Squared
$ 55	61	$ 61	$ (6)	40
$ 92	85	$ 76	$ 16	271
$ 75	95	$ 81	$ (6)	34
$ 96	103	$ 85	$ 11	123
$ 89	127	$ 96	$ (7)	53
$ 86	134	$ 99	$ (13)	178
$ 112	145	$ 104	$ 8	63
$ 95	165	$ 112	$ (17)	297
$ 110	166	$ 113	$ (3)	7
$ 108	173	$ 115	$ (7)	54
$ 138	187	$ 121	$ 17	300
$ 129	194	$ 123	$ 6	33
$ 122	224	$ 134	$ (12)	140
$ 139	243	$ 140	$ (1)	1
$ 140	258	$ 145	$ (5)	25
$ 159	258	$ 145	$ 14	196
$ 144	262	$ 146	$ (2)	5
$ 170	270	$ 149	$ 21	451
$ 161	274	$ 150	$ 11	121
$ 146	277	$ 151	$ (5)	24
$ 143	289	$ 155	$ (12)	134
$ 148	292	$ 155	$ (7)	56
$ 150	292	$ 155	$ (5)	30
$ 169	296	$ 157	$ 12	152
$ 162	297	$ 157	$ 5	25
$ 168	311	$ 161	$ 7	48
$ 159	314	$ 162	$ (3)	9
$ 156	328	$ 166	$ (10)	98
$ 155	334	$ 168	$ (13)	159
$ 165	352	$ 173	$ (8)	57
$ 174	355	$ 173	$ 1	0
$ 177	355	$ 173	$ 4	13
$ 183	356	$ 174	$ 9	87
			SSE =	3,287

a =	-15.60
b =	9.44
c =	0.51
SE =	$ 10.5
R2 =	91%
Bias =	$ 0.0

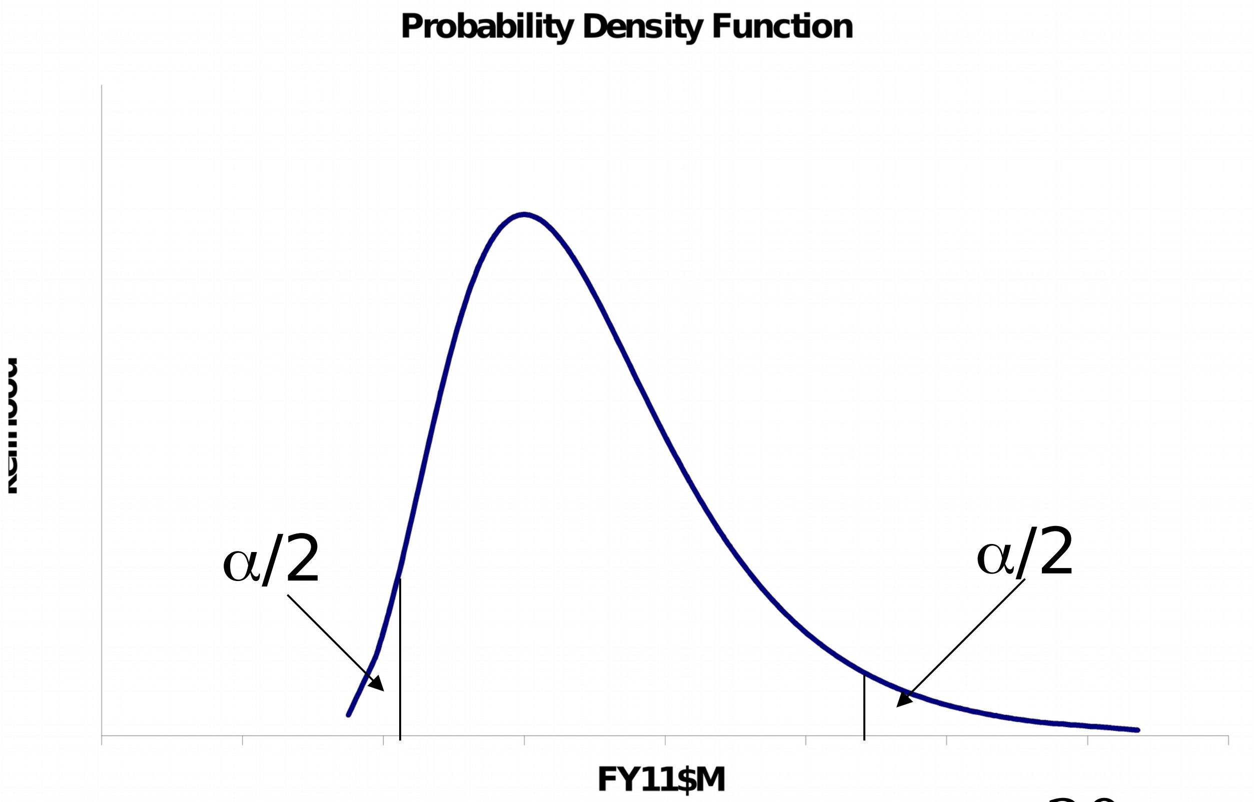
$$Cost = -15.60 + 9.44(Weight)^{0.51}$$

$$SE = \$10.5$$



Now to Determine the $\alpha/2$ Percentiles

- For the lognormal distribution, the $\alpha/2$ percentiles are not symmetric
  - Must determine both the right and left percentiles



Determining the $\alpha/2$ Percentiles (ADDITIVE Errors)

- This is a little tricky with a lognormal distribution, because the lognormal is defined only for *X*>0
- However, we can "shift" the lognormal to arrive at a "pseudo-standardized" lognormal distribution
- Using Excel's "LNORMINV" function, it goes like this…
  - First, remember that we want the "standardized" lognormal $\alpha/2$ or 1-$\alpha/2$ percentile at any given value of *f(x)*, e.g., *f(x*)*
  - For the LHS, we compute the $\alpha/2$ percentile at *f(x*)* with standard deviation *SE*, then we subtract *f(x*)* and divide by *SE* to "standardize" the result

$$\tfrac{\alpha}{2}\ \text{percentile} = \left(\text{LNORMINV}\left(\tfrac{\alpha}{2}, f\left(x^*\right), SE\right) - f\left(x^*\right)\right)/SE$$

  - Similarly, with the RHS, it is…

$$\left(1-\tfrac{\alpha}{2}\right)\text{percentile} = \left(\text{LNORMINV}\left(1-\tfrac{\alpha}{2}, f\left(x^*\right), SE\right) - f\left(x^*\right)\right)/SE$$

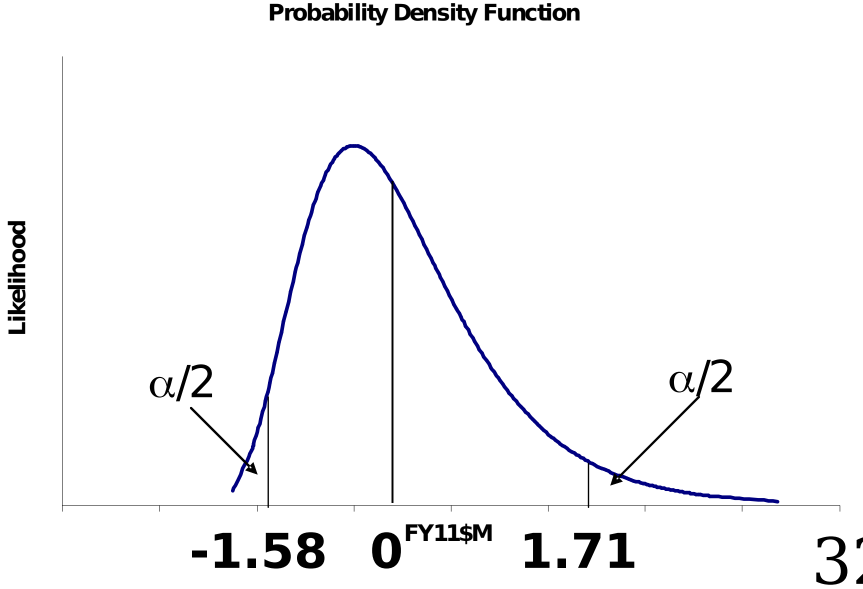
Example

- Suppose *f(x*)* = \$138.7, $\alpha/2$ = 5%, and *SE* = \$10.5
- Then, for the LHS, the $\alpha/2$, or 5th percentile, is computed as

$$5\text{th percentile} = \left(\text{LNORMINV}(0.05, 138.7, 10.5) - 138.7\right)/10.5 = -1.58$$

- And, for the RHS, the ($1-\alpha/2$), or 95th percentile, is computed as

$$95\text{th percentile} = \left(\text{LNORMINV}(0.95, 138.7, 10.5) - 138.7\right)/10.5 = 1.71$$

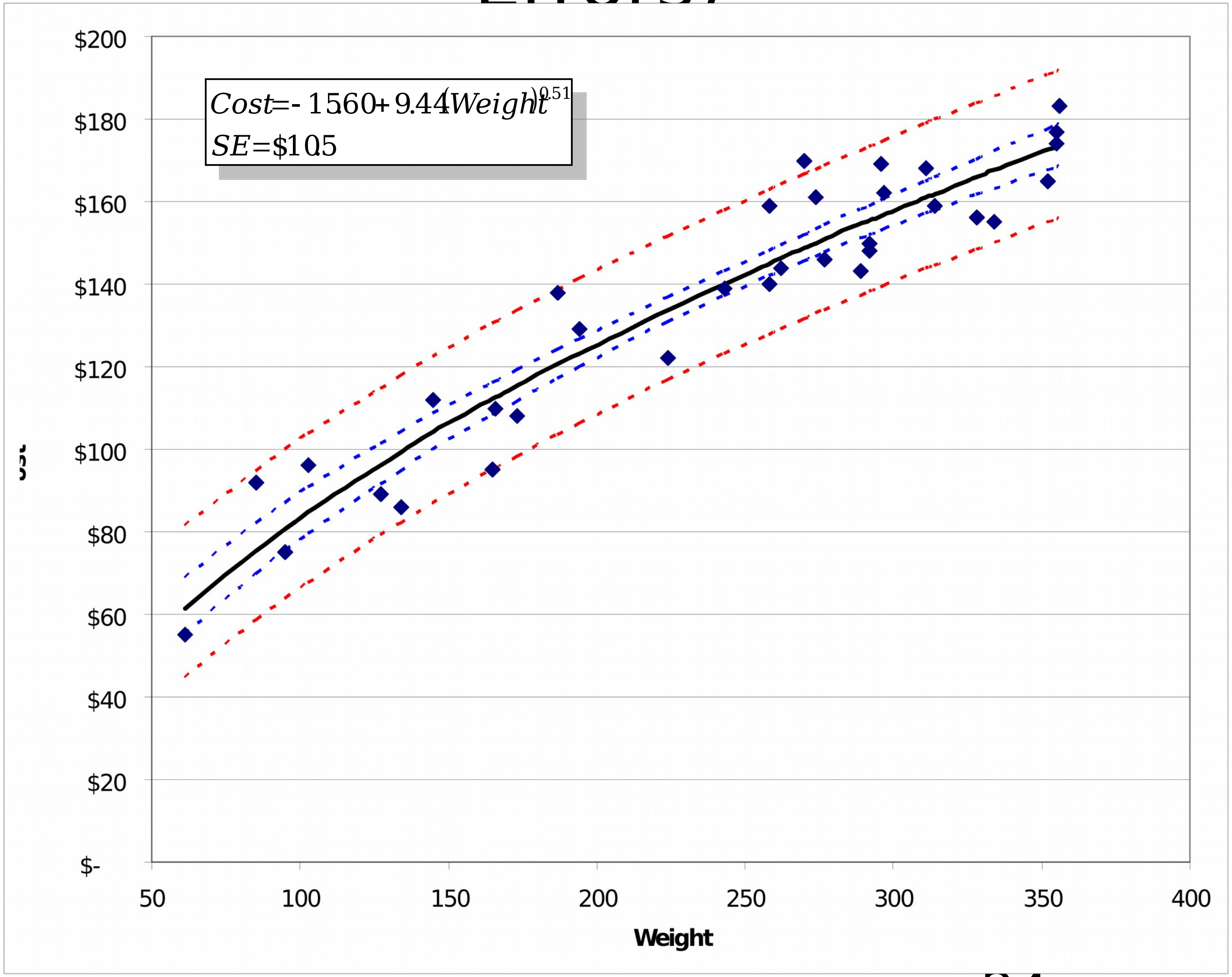


Calculating Approximate CIs and PIs

Cost (Y)	Weight (X)	CER-CI	CER	CER+CI	CER-PI	CER	CER+PI
$ 55	61	$ 55.1	$ 61	$ 68.7	$ 44.6	$ 61.3	$ 81.3
$ 92	85	$ 69.9	$ 76	$ 82.0	$ 58.7	$ 75.5	$ 95.0
$ 75	95	$ 75.5	$ 81	$ 87.0	$ 64.0	$ 80.8	$ 100.1
$ 96	103	$ 79.7	$ 85	$ 90.8	$ 68.0	$ 84.9	$ 104.1
$ 89	127	$ 91.7	$ 96	$ 101.4	$ 79.4	$ 96.2	$ 115.1
$ 86	134	$ 94.9	$ 99	$ 104.3	$ 82.5	$ 99.4	$ 118.1
$ 112	145	$ 99.9	$ 104	$ 108.7	$ 87.3	$ 104.1	$ 122.7
$ 95	165	$ 108.5	$ 112	$ 116.3	$ 95.5	$ 112.2	$ 130.7
$ 110	166	$ 108.9	$ 113	$ 116.7	$ 95.9	$ 112.6	$ 131.1
$ 108	173	$ 111.8	$ 115	$ 119.3	$ 98.6	$ 115.4	$ 133.8
$ 138	187	$ 117.3	$ 121	$ 124.3	$ 103.9	$ 120.7	$ 139.0
$ 129	194	$ 120.0	$ 123	$ 126.8	$ 106.5	$ 123.2	$ 141.5
$ 122	224	$ 130.9	$ 134	$ 137.0	$ 117.1	$ 133.8	$ 152.0
$ 139	243	$ 137.3	$ 140	$ 143.3	$ 123.4	$ 140.2	$ 158.3
$ 140	258	$ 142.0	$ 145	$ 148.2	$ 128.2	$ 145.0	$ 163.1
$ 159	258	$ 142.0	$ 145	$ 148.2	$ 128.2	$ 145.0	$ 163.1
$ 144	262	$ 143.3	$ 146	$ 149.5	$ 129.4	$ 146.3	$ 164.4
$ 170	270	$ 145.7	$ 149	$ 152.1	$ 131.9	$ 148.8	$ 166.9
$ 161	274	$ 146.9	$ 150	$ 153.3	$ 133.1	$ 150.0	$ 168.1
$ 146	277	$ 147.8	$ 151	$ 154.3	$ 134.0	$ 150.9	$ 169.1
$ 143	289	$ 151.2	$ 155	$ 158.1	$ 137.6	$ 154.6	$ 172.7
$ 148	292	$ 152.1	$ 155	$ 159.1	$ 138.5	$ 155.5	$ 173.6
$ 150	292	$ 152.1	$ 155	$ 159.1	$ 138.5	$ 155.5	$ 173.6
$ 169	296	$ 153.2	$ 157	$ 160.4	$ 139.7	$ 156.7	$ 174.8
$ 162	297	$ 153.5	$ 157	$ 160.7	$ 140.0	$ 157.0	$ 175.1
$ 168	311	$ 157.3	$ 161	$ 165.1	$ 144.0	$ 161.1	$ 179.3
$ 159	314	$ 158.1	$ 162	$ 166.0	$ 144.9	$ 161.9	$ 180.2
$ 156	328	$ 161.8	$ 166	$ 170.4	$ 148.8	$ 165.9	$ 184.2
$ 155	334	$ 163.3	$ 168	$ 172.2	$ 150.4	$ 167.6	$ 185.9
$ 165	352	$ 167.8	$ 173	$ 177.7	$ 155.3	$ 172.6	$ 191.0
$ 174	355	$ 168.5	$ 173	$ 178.6	$ 156.0	$ 173.4	$ 191.9
$ 177	355	$ 168.5	$ 173	$ 178.6	$ 156.0	$ 173.4	$ 191.9
$ 183	356	$ 168.8	$ 174	$ 178.9	$ 156.3	$ 173.7	$ 192.2

X-bar	238.5
CER(X-bar)	$ 138.7
S_{XX}	247898.2
SE	$ 10.5
α	10%
1-α	90%
n	33
α/2 %ile	(1.58)
1 - α/2 %ile	1.71

Approximate CI vs. PI (ADDITIVE Errors)

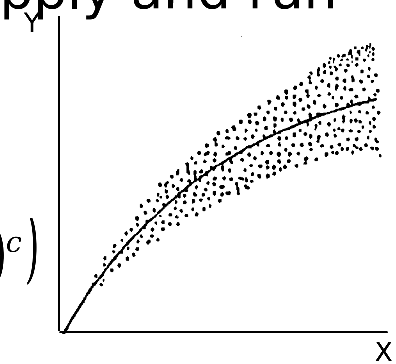


Non-OLS CER Confidence Interval Approximation – MULTIPLICATIVE Errors

- Now suppose we start with a CER of the form $Y=a+bX^c$, with MULTIPLICATIVE errors and a lognormal error specification
- Again we can assume that the same rules apply and run with it
  - Again, what have we got to lose?
- E.g.,

$$Cost = -6301 + 2529(Weight)^{0.38}$$

$$SPE = 8.7\%$$



$$(1-\alpha)CI \approx a+b(x^*)^c \pm (\alpha/2 \text{ percentile}) \times s.p.e.\left(a+b(x^*)^c\right)$$

where

$$s.p.e.\left(a+b(x^*)^c\right) = \hat{\sigma}\sqrt{\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{XX}}} = SPE\sqrt{\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{XX}}}$$

and where

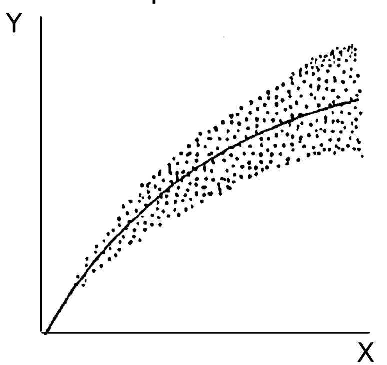
$(\alpha/2 \text{ percentile})$ is the $\alpha/2^{th}$ percentile of the lognormal error distribution

Non-OLS CER Prediction Interval Approximation

- Again it is a simple extension to arrive at the Prediction Interval
- E.g.,

$$Cost = -63.01 + 25.29(Weight)^{0.38}$$

$$SPE = 8.7\%$$



$$(1-\alpha)PI \approx a + b(x^*)^c \pm (\alpha/2 \text{ percentile}) \times s.p.e.\left(a + b(x^*)^c\right)$$

where

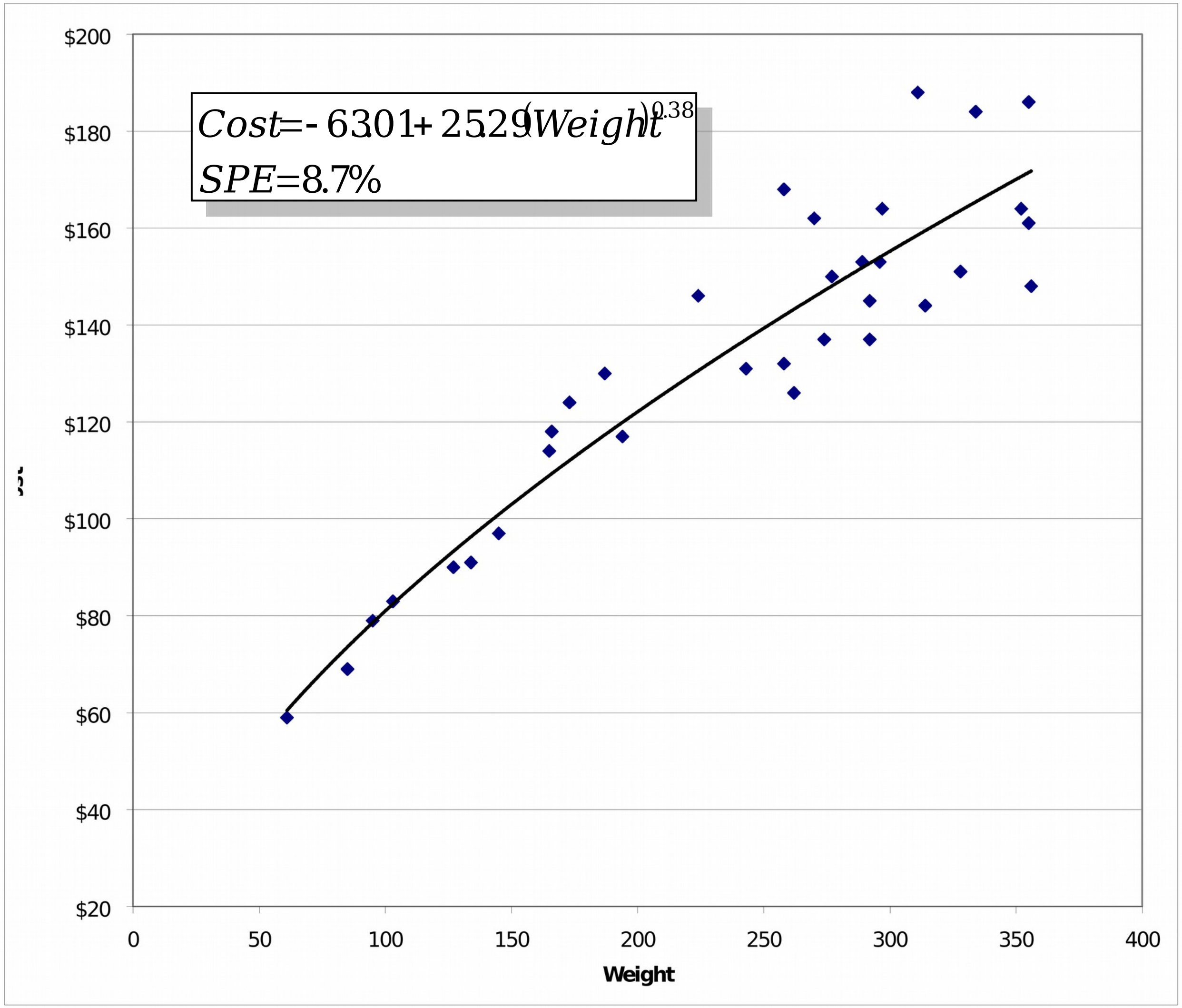
$$s.p.e.\left(a + b(x^*)^c\right) = \hat{\sigma}\sqrt{\frac{n+1}{n} + \frac{(x^* - \bar{x})^2}{S_{XX}}} = SPE \cdot \left(a + b(x^*)^c\right)\sqrt{\frac{n+1}{n} + \frac{(x^* - \bar{x})^2}{S_{XX}}}$$

and where

$(\alpha/2 \text{ percentile})$ is the $\alpha/2^{th}$ percentile of the lognormal error distribution

Example

Cost (Y)		Weight (X)
$	59	61
$	69	85
$	79	95
$	83	103
$	90	127
$	91	134
$	97	145
$	114	165
$	118	166
$	124	173
$	130	187
$	117	194
$	146	224
$	131	243
$	132	258
$	168	258
$	126	262
$	162	270
$	137	274
$	150	277
$	153	289
$	145	292
$	137	292
$	153	296
$	164	297
$	188	311
$	144	314
$	151	328
$	184	334
$	164	352
$	161	355
$	186	355
$	148	356



MULTIPLICATIVE GERM Regression Results

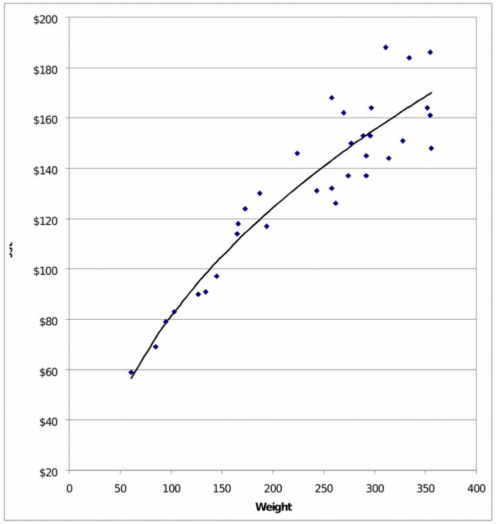
Cost (Y)	Weight (X)	CER	Y-CER	Pct Error	Squared
$ 59	61	$ 57	$ 2	0.04	0.0018
$ 69	85	$ 73	$ (4)	(0.05)	0.0024
$ 79	95	$ 78	$ 1	0.01	0.0001
$ 83	103	$ 83	$ 0	0.00	0.0000
$ 90	127	$ 95	$ (5)	(0.05)	0.0025
$ 91	134	$ 98	$ (7)	(0.07)	0.0051
$ 97	145	$ 103	$ (6)	(0.06)	0.0033
$ 114	165	$ 111	$ 3	0.03	0.0006
$ 118	166	$ 112	$ 6	0.06	0.0033
$ 124	173	$ 114	$ 10	0.08	0.0071
$ 130	187	$ 120	$ 10	0.09	0.0075
$ 117	194	$ 122	$ (5)	(0.04)	0.0018
$ 146	224	$ 133	$ 13	0.10	0.0103
$ 131	243	$ 139	$ (8)	(0.06)	0.0030
$ 132	258	$ 143	$ (11)	(0.08)	0.0062
$ 168	258	$ 143	$ 25	0.17	0.0299
$ 126	262	$ 144	$ (18)	(0.13)	0.0163
$ 162	270	$ 147	$ 15	0.10	0.0107
$ 137	274	$ 148	$ (11)	(0.07)	0.0055
$ 150	277	$ 149	$ 1	0.01	0.0001
$ 153	289	$ 152	$ 1	0.00	0.0000
$ 145	292	$ 153	$ (8)	(0.05)	0.0028
$ 137	292	$ 153	$ (16)	(0.11)	0.0111
$ 153	296	$ 154	$ (1)	(0.01)	0.0001
$ 164	297	$ 155	$ 9	0.06	0.0038
$ 188	311	$ 158	$ 30	0.19	0.0351
$ 144	314	$ 159	$ (15)	(0.10)	0.0091
$ 151	328	$ 163	$ (12)	(0.07)	0.0053
$ 184	334	$ 164	$ 20	0.12	0.0142
$ 164	352	$ 169	$ (5)	(0.03)	0.0009
$ 161	355	$ 170	$ (9)	(0.05)	0.0026
$ 186	355	$ 170	$ 16	0.10	0.0092
$ 148	356	$ 170	$ (22)	(0.13)	0.0167
				SSPE =	0.2284

a =	-63.01
b =	25.29
c =	0.38

SE =	8.7%
R2 =	86%
Bias =	$ 0.0

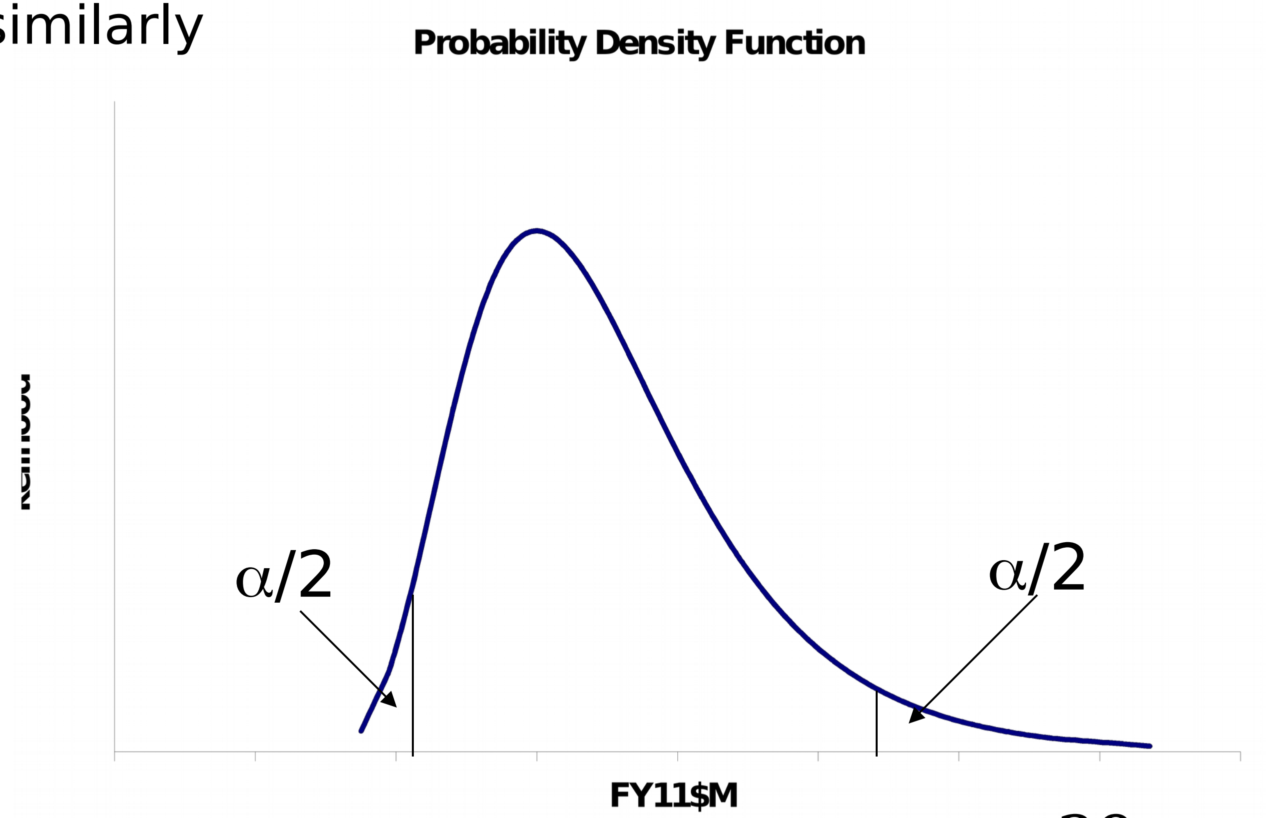
$$Cost = -63.01 + 25.29\,(Weight)^{0.38}$$

$$SPE = 8.7\%$$



Now to Determine the $\alpha/2$ Percentiles

- Again, for the lognormal distribution, the $\alpha/2$ percentiles are not symmetric
  - Must determine both the right and left percentiles similarly



Determining the α/2 Percentiles (MULTIPLICATIVE Errors)

- The technique is similar to how it was done with ADDITIVE errors, but a little different since we are dealing with *SPE* rather than *SE*
- Using Excel's "LNORMINV" function, it goes like this…
  - First, remember that we want the "standardized" lognormal α/2 or 1-α/2 percentile at any given value of *f(x)*, e.g., *f(x*)*
  - For the LHS, we compute the α/2 percentile at *f(x*)* with standard deviation *SPE* × *f(x*)*, then we subtract *f(x*)* and divide by *SPE* × *f(x*)* to "standardize" the result

$$\tfrac{\alpha}{2}\ \text{percentile} = \left(\text{LNORMINV}\left(\tfrac{\alpha}{2}, f\left(x^*\right), SPE \times f\left(x^*\right)\right) - f\left(x^*\right)\right) / \left(SPE \times f\left(x^*\right)\right)$$

  - Similarly, with the RHS, it is…

$$\left(1-\tfrac{\alpha}{2}\right)\ \text{percentile} = \left(\text{LNORMINV}\left(1-\tfrac{\alpha}{2}, f\left(x^*\right), SPE \times f\left(x^*\right)\right) - f\left(x^*\right)\right) / \left(SPE \times f\left(x^*\right)\right)$$

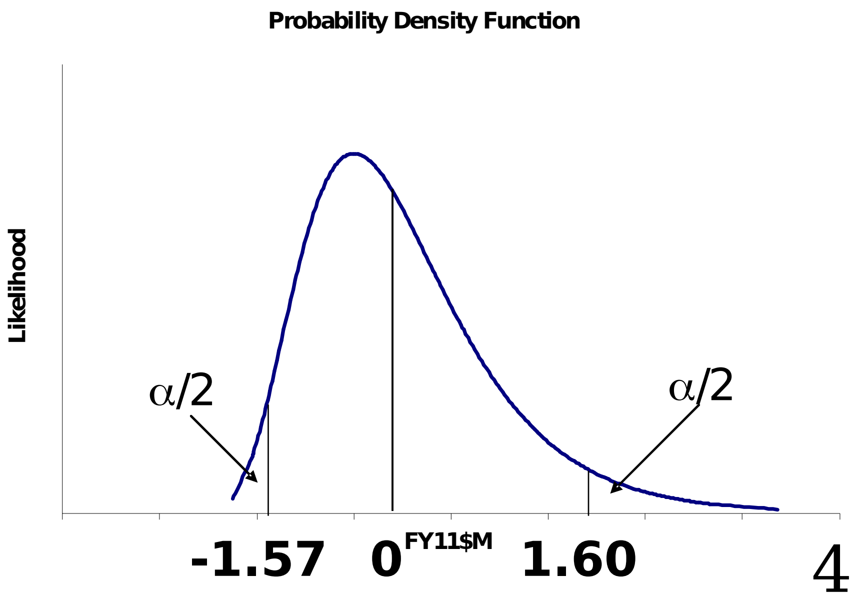
Example

- Suppose *f(x*)* = \$138.2, $\alpha/2$ = 5%, and *SPE* = 8.7%
- Then, for the LHS, the $\alpha/2$, or 5th percentile, is computed as

$$\text{5th percentile} = \left(\text{LNORMINV}(0.05, 138.2, 8.7\% \times 138.2) - 138.2\right) / \left(8.7\% \times 138.2\right)$$
$$= -1.57$$

- And, for the RHS, the $(1-\alpha/2)$, or 95th percentile, is computed as

$$\text{95th percentile} = \left(\text{LNORMINV}(0.95, 138.2, 8.7\% \times 138.2) - 138.2\right) / \left(8.7\% \times 138.2\right)$$
$$= 1.60$$

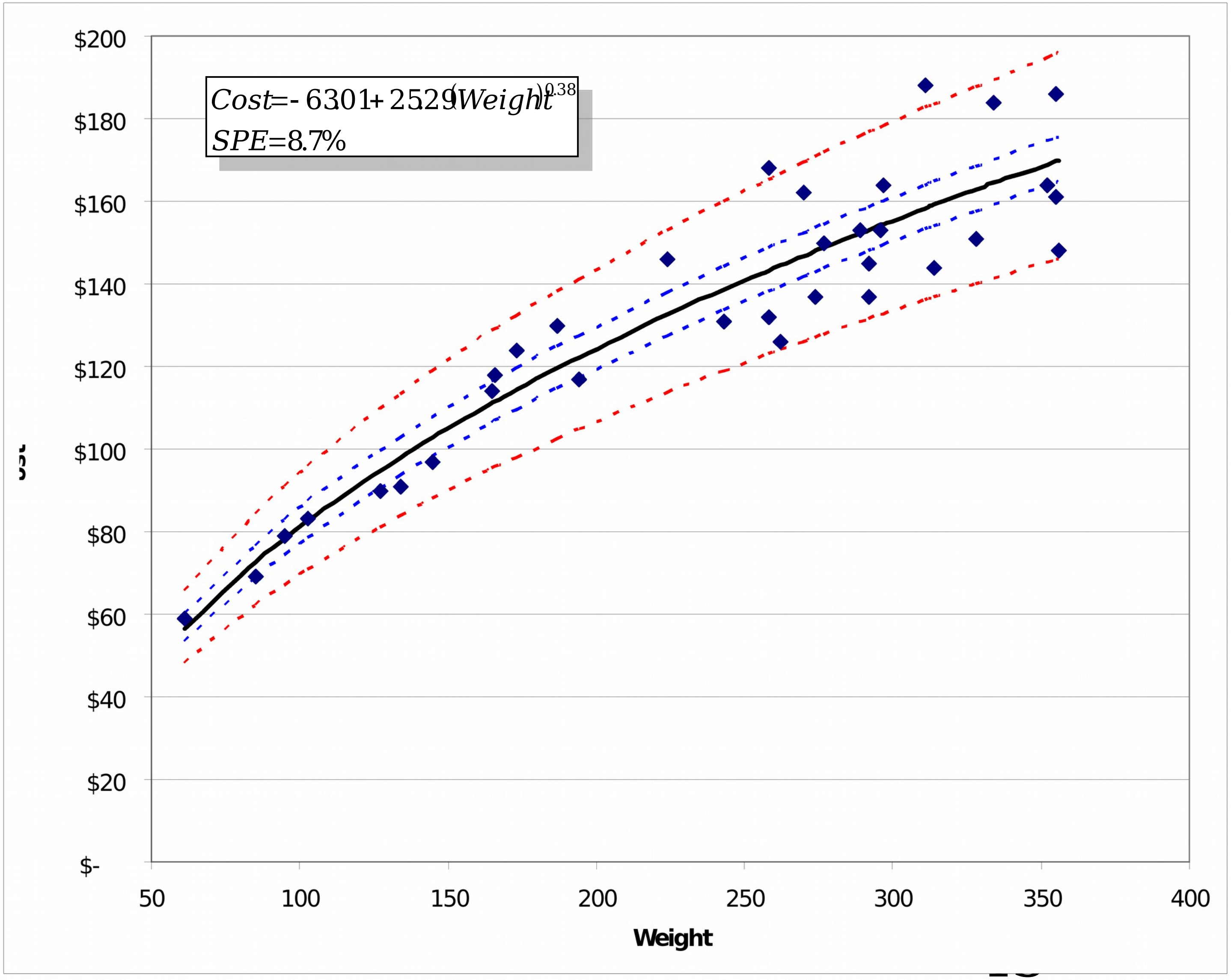


Calculating Approximate CIs and PIs

Cost (Y)	Weight (X)	CER-CI	CER	CER+CI	CER-PI	CER	CER+PI
$ 59	61	$ 53.5	$ 57	$ 60.0	$ 48.2	$ 56.6	$ 65.7
$ 69	85	$ 68.8	$ 73	$ 76.6	$ 62.0	$ 72.6	$ 84.2
$ 79	95	$ 74.5	$ 78	$ 82.7	$ 67.0	$ 78.4	$ 90.9
$ 83	103	$ 78.7	$ 83	$ 87.2	$ 70.7	$ 82.8	$ 95.9
$ 90	127	$ 90.4	$ 95	$ 99.6	$ 81.1	$ 94.8	$ 109.8
$ 91	134	$ 93.6	$ 98	$ 102.9	$ 83.9	$ 98.0	$ 113.5
$ 97	145	$ 98.3	$ 103	$ 107.9	$ 88.1	$ 102.9	$ 119.1
$ 114	165	$ 106.5	$ 111	$ 116.3	$ 95.3	$ 111.2	$ 128.6
$ 118	166	$ 106.9	$ 112	$ 116.7	$ 95.6	$ 111.6	$ 129.1
$ 124	173	$ 109.6	$ 114	$ 119.5	$ 98.0	$ 114.3	$ 132.2
$ 130	187	$ 114.8	$ 120	$ 124.9	$ 102.6	$ 119.6	$ 138.3
$ 117	194	$ 117.3	$ 122	$ 127.5	$ 104.8	$ 122.2	$ 141.2
$ 146	224	$ 127.5	$ 133	$ 138.0	$ 113.7	$ 132.5	$ 153.1
$ 131	243	$ 133.6	$ 139	$ 144.1	$ 119.0	$ 138.6	$ 160.1
$ 132	258	$ 138.2	$ 143	$ 148.8	$ 123.0	$ 143.3	$ 165.4
$ 168	258	$ 138.2	$ 143	$ 148.8	$ 123.0	$ 143.3	$ 165.4
$ 126	262	$ 139.4	$ 144	$ 150.0	$ 124.1	$ 144.5	$ 166.8
$ 162	270	$ 141.7	$ 147	$ 152.4	$ 126.1	$ 146.8	$ 169.5
$ 137	274	$ 142.9	$ 148	$ 153.6	$ 127.1	$ 148.0	$ 170.8
$ 150	277	$ 143.8	$ 149	$ 154.4	$ 127.9	$ 148.9	$ 171.8
$ 153	289	$ 147.2	$ 152	$ 157.9	$ 130.8	$ 152.3	$ 175.8
$ 145	292	$ 148.0	$ 153	$ 158.7	$ 131.6	$ 153.1	$ 176.7
$ 137	292	$ 148.0	$ 153	$ 158.7	$ 131.6	$ 153.1	$ 176.7
$ 153	296	$ 149.1	$ 154	$ 159.8	$ 132.5	$ 154.2	$ 178.0
$ 164	297	$ 149.4	$ 155	$ 160.1	$ 132.8	$ 154.5	$ 178.3
$ 188	311	$ 153.2	$ 158	$ 164.0	$ 136.1	$ 158.3	$ 182.7
$ 144	314	$ 154.0	$ 159	$ 164.8	$ 136.8	$ 159.1	$ 183.6
$ 151	328	$ 157.7	$ 163	$ 168.5	$ 140.0	$ 162.8	$ 187.8
$ 184	334	$ 159.2	$ 164	$ 170.0	$ 141.3	$ 164.4	$ 189.6
$ 164	352	$ 163.8	$ 169	$ 174.6	$ 145.3	$ 168.9	$ 194.9
$ 161	355	$ 164.5	$ 170	$ 175.3	$ 145.9	$ 169.7	$ 195.7
$ 186	355	$ 164.5	$ 170	$ 175.3	$ 145.9	$ 169.7	$ 195.7
$ 148	356	$ 164.8	$ 170	$ 175.6	$ 146.1	$ 169.9	$ 196.0

X-bar	238.5
CER(X-bar)	$ 137.2
S_{XX}	247898.2
SPE	8.7%
α	10%
1-α	90%
n	33
α/2 %ile	(1.57)
□ - α/2 %ile	1.60

Approximate CI vs. PI



Now, Back to CER Uncertainty

- Since we USE CERs to PREDICT costs, then we should also ADJUST the CER uncertainty to reflect PREDICTION ERROR in our Monte Carlo simulations!
- If we simply use the unadjusted CER uncertainty, then we will UNDERSTATE the uncertainty of our predictions
- So, when using a CER in a Monte Carlo simulation, we should adjust the CER uncertainty for prediction error

Adjusting CER Uncertainty for Prediction Error

ADDITIVE Error CER:

$$SE_{adj} = SE\sqrt{\frac{n+1}{n} + \frac{\left(x^{*} - \bar{x}\right)^{2}}{S_{XX}}}$$

MULTIPLICATIVE Error CER:

$$SPE_{adj} = SPE\sqrt{\frac{n+1}{n} + \frac{\left(x^{*} - \bar{x}\right)^{2}}{S_{XX}}}$$

ADDITIVE Error Example

- Suppose we use the following ADDITIVE Error CER in a cost estimate

$$Cost = -1560 + 9.44(Weight)^{0.51}$$

$$SE = \$10.5$$

$$x^* = 200, \quad \bar{x} = 238.5, \quad n = 33$$

$$S_{XX} = 247{,}898.2$$

- Then, for the Monte Carlo simulation, we should model prediction error as

$$Mean\,Cost = -1560 + 9.44(200)^{0.51} = \$125.4$$

$$SE_{adj} = SE\sqrt{\frac{n+1}{n} + \frac{(x^* - \bar{x})^2}{S_{XX}}} = \$10.5\sqrt{\frac{34}{33} + \frac{(200 - 238.5)^2}{247{,}898.2}} = \$10.66$$

MULTIPLICATIVE Error Example

- Now suppose we use the following MULTIPLICATIVE Error CER in a cost estimate

$$Cost = -63.01 + 25.29(Weight)^{0.38}$$

$$SPE = 8.7\%$$

$$x^* = 200, \quad \bar{x} = 238.5, \quad n = 33$$

$$S_{XX} = 247{,}898.2$$

- Then, for the Monte Carlo simulation, we should model prediction error as

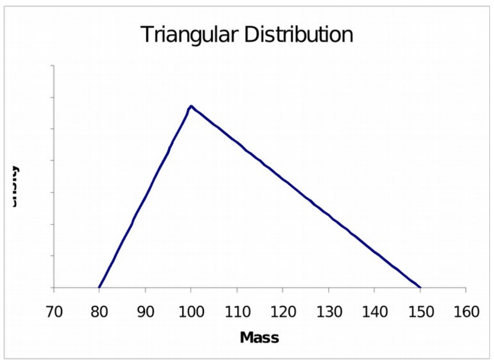
$$Mean\ Cost = -63.01 + 25.29(200)^{0.38} = \$124.3$$

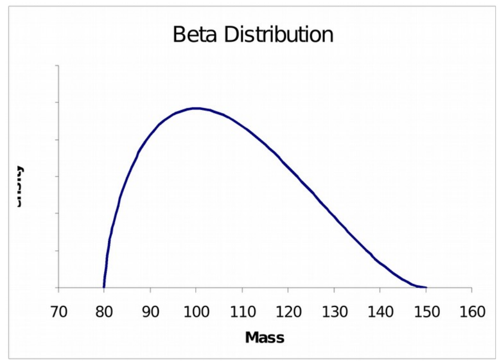
$$SPE_{adj} = SPE\sqrt{\frac{n+1}{n} + \frac{\left(x^* - \bar{x}\right)^2}{S_{XX}}} = 8.7\%\sqrt{\frac{34}{33} + \frac{(200 - 238.5)^2}{247{,}898.2}} = 8.9\%$$

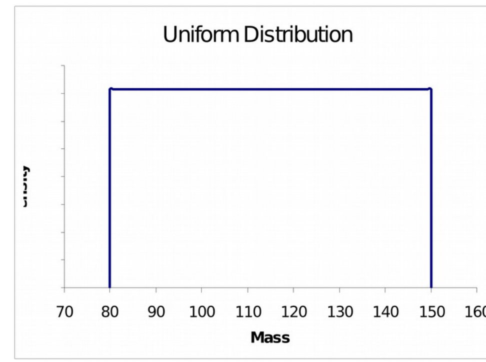
What About Cost Driver Uncertainty?

- So far we’ve said little about this

Triangular? **Beta?** **Uniform?**







- The choice of the shape and the bounds depends on the nature of the uncertainty – **how do we determine this?**
- Next we will spend a little time talking about how to define the shapes and the bounds of the cost drivers

Cost as an Input Variable

- Sometimes, the CER requires the cost of some other aspect of the estimate as an input variable
  - E.g., USCM 7 Ground Support Equipment CER is

$$GSE(FY00\$K) = 9.262(X)^{0.642}$$

where

$$X = \text{Spacecraft bus} + \text{payload RDT\&E cost}$$

- In cases like this, there is no need to model the input variable's uncertainty separately since the bus and payload costs result from CERs which have their own inherent uncertainty

Technical Characteristics as an Input Variable

- However, when the CER requires some technical characteristic such as **size**, **weight**, or **power** as an input variable, then we have to get more creative
  - E.g., the USCM 7 Electrical Power Subsystem (EPS) CER (for T1) is

$$EPST1(FY00\$K) = 112(X)^{0.763}$$

$$\text{where}$$

$$X = \text{EPS weight (kg)}$$

- It is unrealistic to think that the EPS weight is known with certainty
  - Want to model the weight with a probability distribution

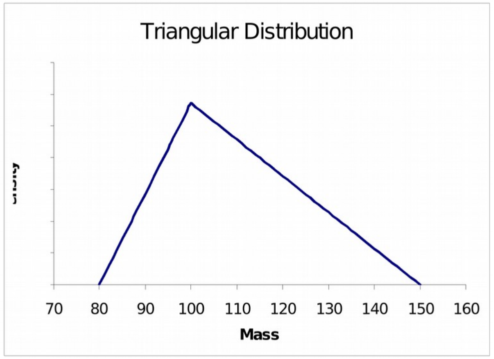
But Which Probability Distribution!?

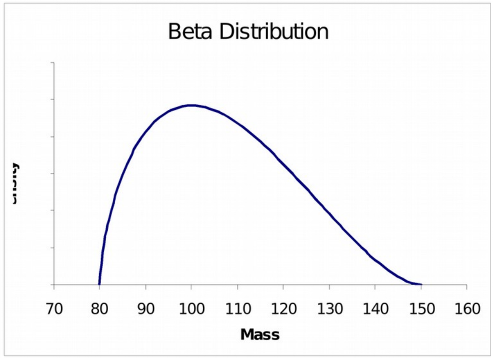
- If you have a database of EPS weights for a variety of spacecraft, then you might be able to develop an empirical distribution
- But it is more likely that you want to put some realistic bounds around **this specific** EPS weight estimate
- For example, suppose the engineering estimate for EPS weight is 100kg
  - Could it be less that that? By how much?
  - Could it be more than that? By how much?
  - Is 100kg the most likely estimate, or could it be more or less with equal probabilities?

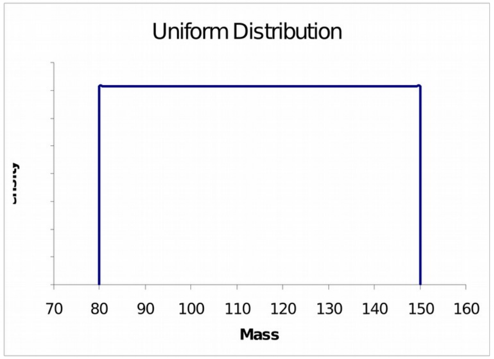
The Usual Suspects

- Usually, it boils down to one of the three shapes shown below

Triangular **Beta** **Uniform**



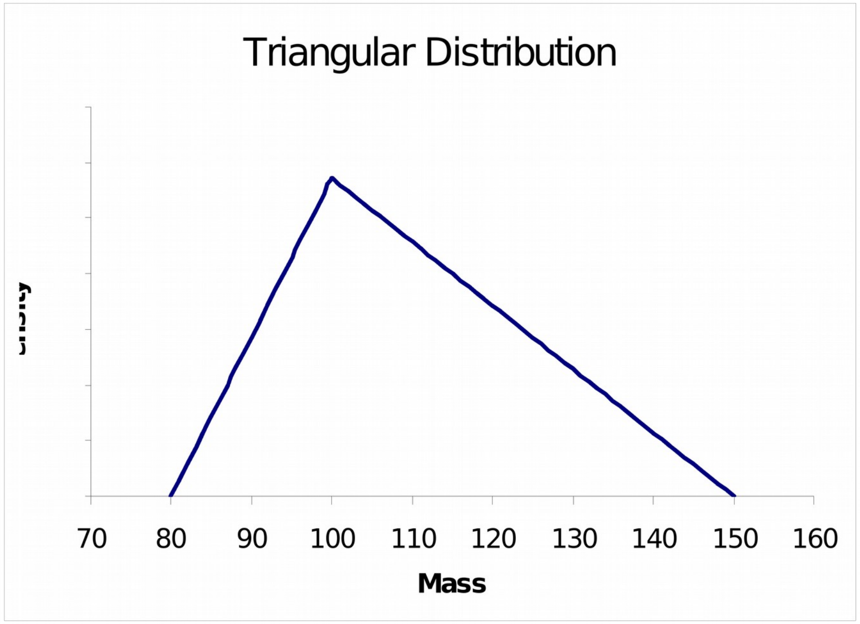




- The **triangular** distribution is used if we think there is a most likely value, bounded on either side
- The **beta** distribution is a smoother variation of the triangular distribution
- The **uniform** distribution is used if the variable can be bounded on either side, with all values in between being equally likely

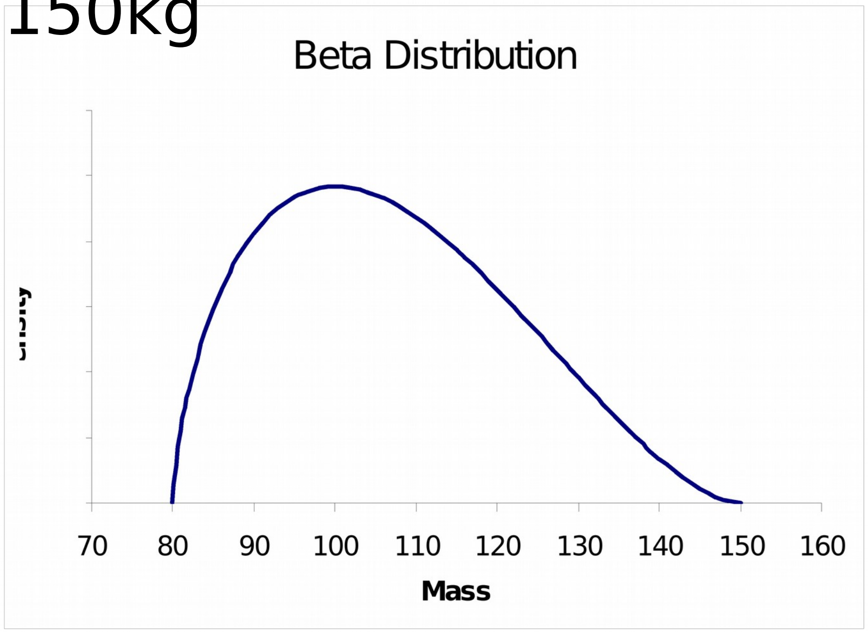
The Triangular Distribution

- The **triangular** distribution is a simple way of describing uncertainty when there is a most likely value with lower and upper bounds
- Example: Suppose you are told that the most likely value is 100kg, but it could be as low as 80kg or as high as 150kg
- The triangle makes a perfectly good probability distribution



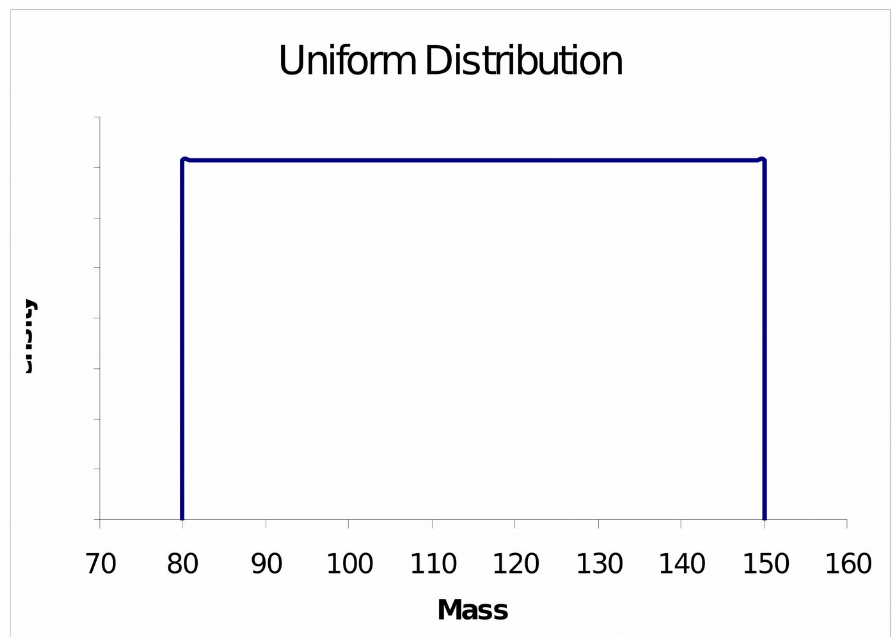
The Beta Distribution

- The **beta** distribution is a slightly more complicated way of describing uncertainty when there is a most likely value with lower and upper bounds
- Example: Suppose you are told that the most likely value is 100kg, but it could be as low as 80kg or as high as 150kg
- Some prefer this depiction because it is smoother



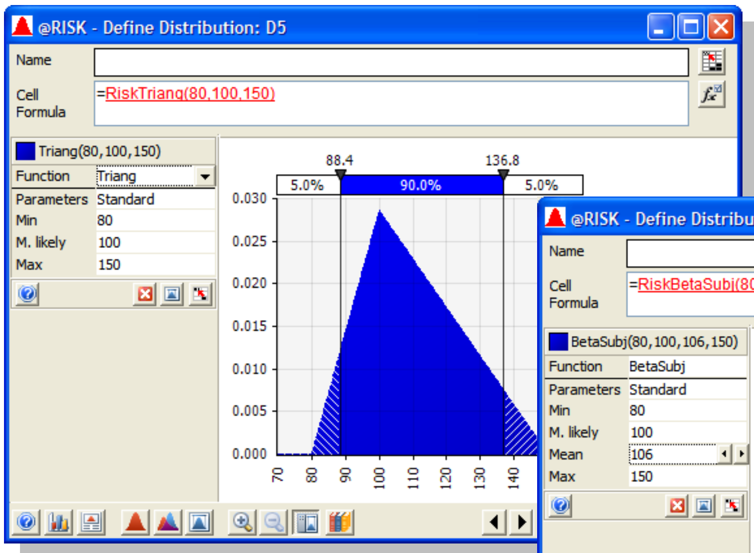
The Uniform Distribution

- The **uniform** distribution is useful when the bounds are known, but there is no ***most likely*** value
- Example: Suppose you are told that the EPS weight could be as low as 80kg or as high as 150kg, with any value in between being equally likely
- This is most often used to represent the uncertainty of learning rates in learning curve calculations

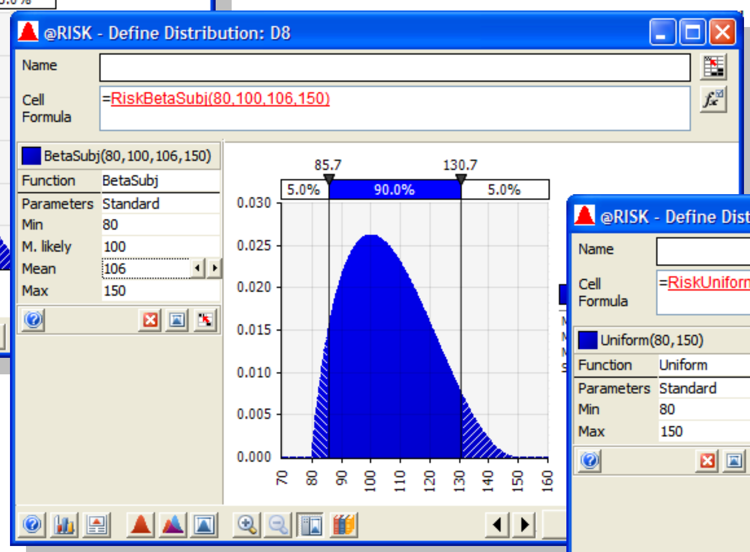


Their Use in Monte Carlo Simulation

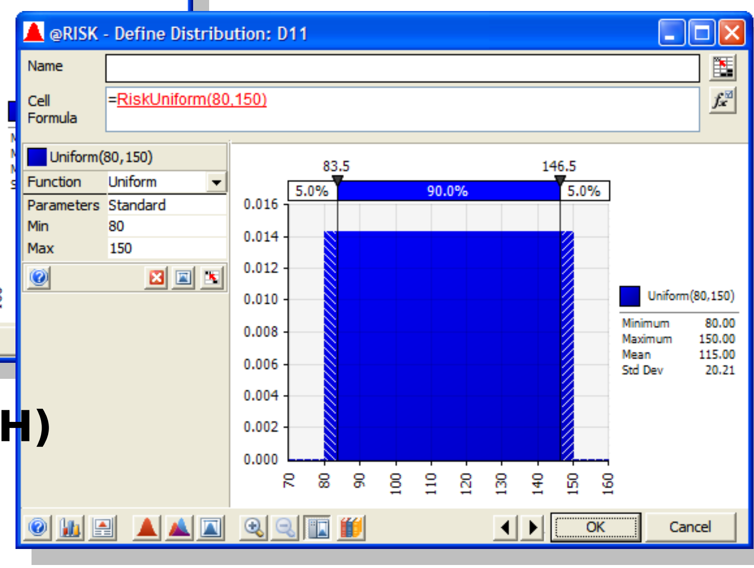
- Modeling these variables in @RISK is trivial…



=RiskTriang(L,ML,H)



=RiskBetaSubj(L,ML,Mean,H)



=RiskUniform(L,H)

But How Do We Determine the Bounds?

- Usually, the most likely, or mean, values can be determined from the program technical description
- But, how do we determine the lower and upper bounds?
- The simplistic answer is to “consult knowledgeable engineers”
- Okay, fine, but how do we extract the information we need from our “knowledgeable engineers?”
  - One engineer may be optimistic, another might be pessimistic
- This is not an easy process, but there are some techniques known as subjective probability assessment that can help us arrive at a good distribution

Subjective Probability Assessment

- There is a large body of knowledge regarding subjective probability assessment
- We will examine some of the more salient points as they relate to building input probability distributions
- Dr. Lionel Galway of RAND wrote on the subject in the *Space Systems Cost Analysis Group Risk Handbook* in 2005
- Much of the following discussion is taken from Dr. Galway's paper
  - Relevant since it was written to address cost estimating

Introduction to Subjective Probability Assessments

- Subjective probabilities are assigned to events on the basis of personal judgment
- Associated with one-time, non-repeatable events
- Subjective probabilities must be consistent with the axioms of probability
  - If there is a 0.8 probability that EPS weight could be less than 100kg, then it must follow that there is a 0.2 probability that EPS weight could exceed 100kg
  - The sum of the probabilities of all mutually exclusive events must be 100%
- To be credible, subjective probabilities should only be assigned by subject matter experts – and not by cost analysts!

Elicitation

- The process of tapping the resources of expert judgment to derive uncertainty bounds when actual data is unavailable is called ***elicitation***
- The general concept is to ask one or more SMEs for the minimum, maximum and most likely values, or some similar combination, then fit a triangular, beta, or uniform distribution using the derived information
  - But, the derived distributions must be checked for consistency (they adhere to the axioms of probability)
  - And, they are subject to human biases, which distort one’s judgment about the uncertainty of their knowledge
  - However, the more knowledgeable the SME, the less the effect of bias

Common Human Biases in SPA

- Availability
  - The tendency to overestimate the probability of events that are easy to recall
- Representativeness
  - Judging probability of events by focusing on characteristics (possibly irrelevant) in which they resemble other events
- Anchoring and Adjustment
  - An initial assessment of a value biases the final assessment toward that value by constraining subsequent adjustment of the assessment in the light of new evidence
- Overconfidence
  - ***Under***estimation of uncertainty about a quantity

Best Practices in Elicitation of SPA

- Use multiple experts, if possible
  - If program engineers are used, independent engineers should also be included if feasible
- Ask the expert to provide, at a minimum, upper, lower, and most likely values
  - Push the expert to think of reasons why the range could be larger – especially on the high side
  - Ask for the endpoints first, before asking for the most likely value, to mitigate effects of anchoring
- Assume the minimum and maximum values provided are actually the 5^{th} and 95^{th} percentiles
  - Adds more spread to the distribution and helps counteract over-optimism

Best Practices in Elicitation of SPA

- Elicit at least two additional percentiles (in addition to minimum, maximum and most likely)
  - For example, ask for the 20th and 80th percentiles
  - Their answers may not be consistent with their earlier answers – if not, try to resolve
  - Elicit percentiles in multiple ways to help check for and diagnose bias
- Provide feedback to the SME about the results of the elicitation
  - Visual aids may help the SME resolve some of his/her biases
- Document the elicitation and archive the data for future studies and comparison to actual results

A Monte Carlo Simulation Demo

Assessing input/CER uncertainty and computing the prediction error

Summary

- What we have covered today:
  - Sources of CER uncertainty
  - Sources of cost driver uncertainty
  - Other types of uncertainty
  - Prediction error vs. standard error
  - Subjective probability assessment

Reading and Homework

- For this week:
  - Read Chapter 2 (Section 2.3) of Garvey
  - Re-read Chapter 4 (Section 4.5) of Garvey
  - Read pages 338 – 339 (Subjective Probability Assessment) of Garvey
  - Do Homework Assignment 4 in Sakai
    - Garvey Chapter 4, Exercises 22 and 24